DOCUMENT RESUME

ED 201 471 SE 034 609

AUTHOR	Hirata, Li Ann
TITLE	Core Geometry Manual.
INSTITUTION	Hawaii Univ., Honolulu. Curriculum Research and Development Group.
SPONS AGENCY	Hawaii State Dept. of Education, Honolulu. Office of Instructional Services.
PUB DATE	80
NOTE	187p.
EDRS PRICE	MF01/PC08 Plus Postage.
DESCRIPTORS	Course Descriptions; Curriculum Development; Geometric Concepts; *Geometry; Instructional Materials; *Lesson Plans; Mathematical Concepts; Mathematics Curriculum; *Mathematics Instruction; Problem Solving; Secondary Education; *Secondary School Mathematics; *Teaching Guides; Teaching Methods; Tests; *Worksheets

ABSTRACT

Core Geometry is a course offered in the Option Y sequence of the high school mathematics program described by the Hawaii State Department of Education's guidelines. The emphasis of this course is on the general awareness and use of the relationships among points, lines, and figures in planes and space. This sample course is based on the consumable worktext "Geometry" by Ruth E. M. Wong. There are 15 learner objectives for the course, with many of the lessons designed for classwork rather than homework. This document presents a course outline and suggested time schelule, with the bulk focusing on teacher commentary. Masters for worksheets, quizzes, and a unit test designed for the outlined teaching guide are provided. (MP)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

U.S. DEPARTMENT OF HEALTH
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

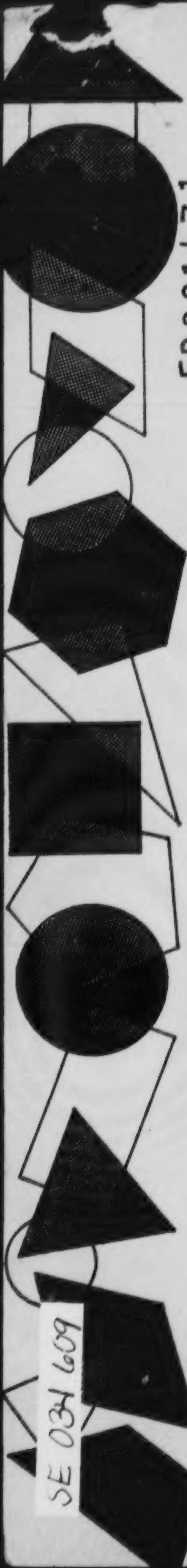
"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

P. Izumo

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE

ED201471



CORE
GEOMETRY
MANUAL

SE 034 609

ACKNOWLEDGMENTS

Gratitude is extended to Linda Ito and Juliette Tom of Moanalua High School who participated in the pilot testing of the module. Appreciation is also extended to Ron Toma, the Central Oahu District mathematics educational specialist, who facilitated the conduct of the pilot test.

CORE GEOMETRY MANUAL

Written and developed under the direction of Nancy C. Whitman

Writer and Developer: Li Ann Hirata

Consultant: Ruth E. M. Wong

Graphic Layout and Typing: Gloria Chun, Arleen Kawabata, Kimberly Kihara, Elizabeth Jane Manuel, Linda Saito, and Cynthia Suehiro

Developed by

Curriculum Research and Development Group,
College of Education,
University of Hawaii
for the
Department of Education

Funded by
Title IV-C, Elementary and Secondary Act

1980

INTRODUCTION

Core Geometry is a course offered in the Option y sequence of the high school mathematics program described in the Hawaii State Department of Education's Mathematics Program Guide 7-12. According to the guide, the emphasis of the course is on the general awareness and use of the relationships among points, lines, and figures in planes and space. It is less theoretical than Geometry A and Geometry B.

The Mathematics Program Guide lists fifteen learner objectives for the course. These objectives indicate the minimum course content for a Core Geometry course. They are listed below:

1) Uses angle relationships such as vertical angles, complementary angles, and supplementary angles to solve problems.

2) Uses concepts relating to triangles such as the sum of the measures of the angles of a triangle is 180°.

3) Investigates and uses the Pythagorean theorem.

4) Is aware of and uses the notion of "congruent figures."

5) Uses the notion of "Perpendicularity" to solve problems.

6) Discovers and uses conditions that imply parallelism; also discovers and uses conditions that are implied by parallelism.

7) Investigates and uses properties of quadrilaterals.

8) Is aware of the notion of "similar figures."

9) Uses ratio and proportion to solve problems involving similarity.

10) Uses formulas to find the areas and perimeters of polygonal regions.

11) Uses formulas to find the circumference and area of a circle.

12) Investigates and uses formulas to find the volumes of common geometric solids (e.g., rectangular solids, spheres, cylinders, and cones).

13) Uses translations, rotations, reflections, expansions, and contractions.

14) Uses the notion of "symmetry."

15) Is aware of how statements in mathematics, in this case, geometry, can be organized into a deductive system, that is, a system consisting of undefined terms, defined terms, axioms, theorems, and relationships.

Part I of this module presents a brief description of materials that may be used for this course. Part II offers a sample course, using one of the materials described.

Part I: Available Materials

The number of available materials for teaching a Core Geometry course is limited. Most available textbooks tend to be written for mathematically talented students. Recently, however, the need for geometry materials for the typical student in high school has been recognized. A brief description of some of the materials currently available follows.

Jurgensen, R. C., and Brown, R. G. Basic Geometry. Boston: Houghton Mifflin Co., 1978. $8.49 (Teacher's Edition: $9.96)

This textbook was designed for students who would normally have difficulty with a formal geometry course. It provides a simple introduction to the fundamentals of Euclidean geometry. The lessons are short and easy to read, with many interesting activities for the students. There are lots of practical applications and many hands-on activities.

All of the learner objectives for the Core Geometry course are covered except for the two objectives on transformations. Supplementary materials on transformations would have to be used if this text is adopted for the course.

In addition to the student text, the publisher provides a teacher's annotated edition, a solution key, and a booklet of progress tests.

Lange, M. Geometry in Modules: An Informal Course. Menlo Park, CA: Addison-Wesley Publishing Co., 1975. Each of modules A-D: $2.13 (Teacher's Edition: $7.44)

Geometry in Modules is a set of four softbound texts that provide a complete course in high school geometry. It uses a discovery approach and is designed to reach students of varying mathematical maturity. It claims to be particularly effective with students who would have been excluded from the study of geometry because a formal course is too difficult.

All of the learner objectives for the Core Geometry course are covered. In fact, the authors claim that the text includes all the properties and problem-solving techniques normally found in a high school geometry course and more.

A teacher's manual is also available from the publisher. The manual provides an answer key to the student text and a commentary. Sample chapter tests are provided too.

O'Daffer, P., and Clemens, S. Laboratory Investigations in Geometry. Menlo Park, CA: Addison-Wesley Publishing Co., 1976.

This workbook is designed to provide experience in the inductive discovery of geometry relationships. It can complement a formal or an informal geometry course.

Whitman, N. C. Experiments for Secondary School Geometry. Honolulu, HI: Research Corporation of the University of Hawaii, 1972.

This is a booklet containing forty experiments in geometry. The experiments are designed to enrich and supplement a geometry class. The topics include: congruence, geometric inequalities, parallels, areas of regions, similarity, circles, constructions, polygons and area of circles, and solids.

Wong, R. E. M. Geometry. Honolulu, HI: Research Corporation of the University of Hawaii, 1980.

This consumable worktext was designed to encourage student exploration and experimentation with selected geometric ideas. Students have the opportunity to become actively involved in finding out results for themselves. In this approach there are many opportunities for the students to see and to become involved in problem solving. They are often asked to look for patterns in their data, to make and verify conjectures, and to generalize from specific instances.

All of the learner objectives for the Core Geometry course are covered, except for the objectives on using the notion of "symmetry." The eight units of the text include: Congruence, Inequalities, Parallelism and Other Relations in Planes and Space, Similarity, Polygons and Polyhedra, Circles and Spheres, Areas and Volumes, and Transformations.

Part II: A Sample Course

This is a Core Geometry course based on the consumable worktext, Geometry by Ruth E. M. Wong. The eight units of the text include: Congruence, Inequalities, Parallelism and Other Relations in Planes and Space, Similarity, Polygons and Polyhedra, Circles and Spheres, Areas and Volumes, and Transformations. As stated in the preface of the text, the materials are designed to encourage student exploration and experimentation with selected geometric ideas. Students are actively involved in finding out results for themselves. In this approach there are many opportunities for the students to see and to become involved in problem solving. They are often asked to look for patterns in their data, to make and verify conjectures, and to generalize from specific instances.

Many of the lessons in the text are designed for classwork rather than homework. If students have difficulty with the reading or with a problem, they will be less likely to "give up" if the teacher or a classmate is there to offer help. It is recommended that the teacher encourage students to read and help them in their reading by using methods suggested by experts like Richard Earle (in Teaching Reading and Mathematics) and Dorothy Piercy (in Reading Activities in Content Areas, An Ideabook for Middle and Secondary School).

Teacher-led discussion is essential throughout the study of the course. Discussion gives students the opportunity to develop language and communication skills. Polya's four steps to problem-solving strategies (see pages 15-22 in the Mathematics Program Guide) should also be emphasized in the discussion.

The text does not have a summary for each unit of glossary. Rather, the author suggests that the students be asked to make their own lists of definitions and conclusions for references as they work through each unit. This means that students themselves must write accurate and complete statements instead of simply reading them as would occur if provided with a summary.

Because answer keys tend to emphasize "right" answers rather than promoting thought and discussion, an annotated teacher's edition of the text or answer key booklet is not provided. Teachers should be aware that there may be more than one answer for some questions.

COURSE OUTLINE AND SUGGESTED TIME SCHEDULE. The course outline and a suggested time schedule are shown on the next pages. The time schedule is based on 45-minute periods which meet daily. It should be adapted to individual situations.

TEACHER COMMENTARY. The Teacher Commentary section follows the Course Outline and Suggested Time Schedule. This section provides an outline of objectives, materials needed, and teaching suggestions for each unit in the text. The "Suggestions for Teaching" include notes about the lessons under the headings: Vocabulary, Symbols, Materials Needed, and Comments.

The "Vocabulary" lists the new terms for each set. The teacher should point out these new terms to the students and help them to pronounce the words correctly, to learn their meanings, and to use them properly. Students should be encouraged to use the new terms whenever possible.

A list of new symbols for the set are given under the second heading. The teacher should spend time familiarizing the students with the new symbols.

"Materials Needed" is a list of materials to be used for each set. These are supplies other than paper and pencil.

Comments on instructional strategies are given under the last heading. These notes point out ideas that should be emphasized in a class discussion as well as provide suggestions to encourage problem-solving behavior by the students. In many cases, problem-solving strategies are highlighted in the margins of the commentaries. However, the teacher should try to promote problem-solving behavior and skills at other times as well.

QUIZZES AND TESTS. Sample quizzes and tests are provided for each unit. They follow directly after the commentary of each unit. The quizzes are usually between 5 and 15 minutes in length. The tests are about 45 minutes long. Supplementary worksheets are also provided. Notations are given in the "Suggestions for Teaching" to indicate where these quizzes, tests, and supplementary worksheets may be best assigned.

COURSE OUTLINE AND SUGGESTED TIME SCHEDULE FOR CORE GEOMETRY

Introduction (1 day)

Make announcements.
Distribute texts.
Assign worksheet, "Reading the Text," and discuss the worksheet with the class.
Assign Unit I, Set 1.

Unit I: Congruence (17 days)

Day 1 Class discussion: Set 1
Assign Set 2

Day 2 Class discussion: Set 2
Class discussion: Set 3
Assign Set 3

Day 3 Class discussion: Set 3
Class discussion: Set 4
Assign Set 4, pages 16 - 20

Day 4 QUIZ 1
Class discussion: Set 4
Assign Set 4, pages 21 - 23

Day 5 Class discussion: Set 4
Assign Set 4, pages 24 - 25

Day 6 Class discussion: Set 4
Class discussion: Set 5
Assign Set 5, pages 26 - 30, #1 - 16

Day 7 QUIZ 2
Class discussion: Set 5
Assign Set 5, pages 30 - 34

Day 8 Class discussion: Set 5
Assign Set 6, #1 - 18

Day 9 QUIZ 3
Class discussion: Set 6
Assign rest of Set 6 and discuss as a class.

Day 10 Class discussion: Set 7
Assign Set 7

Day 11 Class discussion: Set 7
Class discussion: Set 8
Assign Set 8, #1 - 5

Day 12 Class discussion: Set 8
Assign Set 8, #6 - 12

Day 13 Class discussion: Set 8
Class discussion: Set 9 pages 45 - 48
Assign Set 9, pages 49 - 51

Day 14 Class discussion: Set 9
Assign Set 9, #1 - 3 and Review Worksheet, #1 - 8

Day 15 Class discussion: Set 9 and Review Worksheet
Assign Set 9, #4 - 6 and Review Worksheet, #9 - 13

Day 16 Class discussion: Set 9 and Review Worksheet
QUIZ 4
Assign Set 9, #7 & 8 and discuss as a class.

Day 17 UNIT I TEST
Homework: Unit II, Set 1, #1 - 23

Unit II: Inequalities (10 days)

Day 1 Class discussion: Set 1
Class discussion: Set 2
Assign rest of Set 1 and Set 2

Day 2 Class discussion: Sets 1 & 2
Class discussion: Set 3
Assign Set 3

Day 3 Class discussion: Set 3
QUIZ 1
Assign Set 4

Day 4 Class discussion: Set 4
Class discussion: Set 5
Assign Set 5

Day 5 Class discussion: Set 5
Class discussion: Set 6
Assign Set 6, #1 - 4

Day 6 Class discussion: Set 6
Assign Set 6, #5 - 8 and Take-home Test

Day 7 Class discussion: Set 6
Class discussion: Set 7, pages 73 - 78 (before Theorem 7)
Assign Review Worksheet, #1 - 6

Day 8 QUIZ 2
Class discussion: Review Worksheet
Class discussion: Set 7, pages 78 - 81
Assign Set 7, #1 - 3 and Review Worksheet, #7 - 10

Day 9 Class discussion: Set 7 and Review Worksheet
Assign Set 7, #4 - 6 and discuss as a class.

Day 10 UNIT II TEST
Homework: Unit III, Set 1, #1 - 4

Unit III: Parallelism and Other Relations in Planes and Space (10 days)

Day 1 Class discussion: Set 1
Assign rest of Set 1 and discuss as a class.

Day 2 Class discussion: Set 2
Assign Set 2
Class discussion: Set 2

Day 3 Assign Set 3, #1 - 6
Class discussion: Set 3
Assign rest of Set 3

Day 4 Class discussion: Set 3
Class discussion: Set 4
Assign Set 4, #1 - 5

Day 5 QUIZ 1
Class discussion: Set 4
Assign rest of Set 4 and Set 5, #1 - 4

Day 6 Class discussion: Sets 4 & 5
Assign Set 5, #5 - 10

Day 7 Class discussion: Set 5
Class discussion: Set 6, pages 98 - 101
Assign Review Worksheet, #1 - 4

Day 8 QUIZ 2
Class discussion: Review Worksheet
Class discussion: Set 6, pages 102 - 105
Assign Set 6, #1 - 3 and Review Worksheet, #5 - 8

Day 9 Class discussion: Review Worksheet and Set 6
Assign Set 6, #4 - 6 and discuss as a class.

Day 10 UNIT III TEST
Homework: Unit IV, Set 1, #1 - 10

Unit IV: Similarity (11 days)

Day	
Day 1	Class discussion: Set 1 Assign rest of Set 1
Day 2	Class discussion: Set 1 Class discussion: Set 2 Assign Set 2 Assign Set 3, #1 & 2
Day 3	Class discussion: Sets 2 & 3 Assign Set 3, #3 - 10
Day 4	Class discussion: Set 3 Assign rest of Set 3
Day 5	Class discussion: Set 3 Class discussion: Set 4 Assign Set 4
Day 6	QUIZ 1 Class discussion: Set 4 Assign Set 5
Day 7	Class discussion: Set 5 Class discussion: Set 6, pages 126 - 129
Day 8	Class discussion: Set 6, pages 130 - 134 Assign Set 6, #1 & 2 and Review Worksheet, #1 - 5
Day 9	Class discussion: Set 6 Assign Set 6, #3 - 5 and Review Worksheet, #6 - 12
Day 10	Class discussion: Set 6 and Review Worksheet Assign Take-home Test
Day 11	UNIT IV TEST Homework: Unit V, Set 1, #1 - 3

Unit V: Polygons and Polyhedra (10 days)

Day	
Day 1	Class discussion: Set 1 Assign rest of Set 1
Day 2	Class discussion: Set 1 Assign Set 2, #1 - 12
Day 3	Class discussion: Set 2 Assign rest of Set 2 and Set 3
Day 4	Class discussion: Sets 2 and 3 Class discussion: Set 4 Assign Set 4
Day 5	QUIZ 1 Class discussion: Set 4 Class discussion: Set 5 Assign Set 5
Day 6	Class discussion: Set 5 Assign Set 6
Day 7	QUIZ 2 Class discussion: Set 6 Assign Review Worksheet, #1 - 4
Day 8	Class discussion: Review Worksheet Class discussion: Set 7 Assign Set 7 for in-class work Assign Review Worksheet, #5 - 9
Day 9	Class discussion: Review Worksheet Assign Take-home Test
Day 10	UNIT V TEST Homework: Unit VI, Set 1

Unit VI: Circles and Spheres (9 days)

Day 1 Class discussion: Set 1
Class discussion: Sets 2 and 3
Assign Set 2 and Set 3, #1 & 2

Day 2 Class discussion: Sets 2 and 3
Assign Set 3, #3 - 16

Day 3 QUIZ 1
Class discussion: Set 3
Assign rest of Set 3 and
Set 4, #1 & 2

Day 4 Class discussion: Sets 3 and 4
Assign rest of Set 4

Day 5 Class discussion: Set 4
Class discussion: Set 5
Assign Set 5 and Set 6, #1 - 8

Day 6 QUIZ 2
Class discussion: Sets 5 and 6
Assign rest of Set 6 and
Review Worksheet, #1 - 4

Day 7 Class discussion: Set 6 and
Review Worksheet
Assign rest of Review Worksheet

Day 8 Class discussion: Review
Worksheet
Assign Take-home Test

Day 9 UNIT VI TEST
Homework: Unit VII, Set 1,
#1 - 5

Unit VII: Areas and Volumes (10 days)

Day 1 Class discussion: Set 1
Assign rest of Set 1

Day 2 Class discussion: Set 1
Class discussion: Set 2
Assign Set 2

Day 3 Class discussion: Set 2
Class discussion: Set 3
Assign Set 3, #1 - 9

Day 4 QUIZ 1
Class discussion: Set 3
Assign Set 3, #10 - 17

Day 5 Class discussion: Set 3
Class discussion: Set 4
Assign Set 4

Day 6 Class discussion: Set 4
Class discussion: Set 5
Assign Set 5, #1 - 13

Day 7 QUIZ 2
Class discussion: Set 5
Assign Set 5, #14 - 20 and
Review Worksheet, #1 - 3

Day 8 Class discussion: Set 5
Assign rest of Review
Worksheet

Day 9 Class discussion: Review
Worksheet

Day 10 UNIT VII TEST
Homework: Unit VIII, Set 1,
#1 - 6

Unit VIII: Transformations (8 days)

Day 1 Class discussion: Set 1
Assign rest of Set 1

Day 2 Class discussion: Set 1
Class discussion: Set 2
Assign Set 2

Day 3 Class discussion: Set 2
Class discussion: Set 3
Assign Set 3

Day 4 QUIZ 1
Class discussion: Set 3
Assign Set 4

Day 5 Class discussion: Set 4
Assign Review Worksheet

Day 6 QUIZ 2
Class discussion: Review Worksheet
Assign Set 5

Day 7 Class discussion: Set 5
Review

Day 8 UNIT VIII TEST

TEACHER COMMENTARY

Reading the Text

Objective: To become familiar with the text.

Comments: After the texts have been distributed to the students, assign the worksheet titled, "Reading the Text." Allow time for the students to do the worksheet on their own before having them discuss the answers as a class. For some of the students, the exercise will be a real eye-opener!

Throughout the study of the course, students should be encouraged to use the features of the text such as the Table of Contents and the Index. For example, refer the students to the Index if they need a review on the construction of the perpendicular bisector of a line segment.

Students should also be aware of the use of other features of the text such as underlining and diagrams. Point out and discuss these features as they appear in the text.

Reading the Text

Name ____________________

Date ____________________

Referring to your text, answer the following questions:

____________________ 1. What is the title of the text?

____________________ 2. Who is the author?

____________________ 3. How recent is the text?

____________________ 4. Where is the Table of Contents?

____________________ 5. What information is in the Table of Contents?

____________________ 6. Where is the Index?

____________________ 7. How is the Index used?

____________________ 8. How many units does the book contain?

____________________ 9. How is each unit subdivided?

____________________ 10. On what page does the unit on similarity begin?

____________________ 11. In what unit can you find out about circles?

____________________ 12. On what page can you find information about "icosahedron"?

____________________ 13. On what pages can you find information on "parallelograms"?

____________________ 14. a. On page 27, what words are underlined?

____________________ b. Why do you think these words are underlined?

____________________ 15. On page 39, find the word "supplementary." According to the book, when are angles supplementary?

____________________ 16. On page 6, the symbol, "$\overline{AB}$," is written. What does it mean?

Unit I: Congruence

Objectives

(1) To identify congruent figures.

(2) To name congruent triangles and their corresponding parts.

(3) To verify and use the SSS, SAS, and ASA Postulates to prove triangles congruent.

(4) To do the following basic constructions with a compass and a straightedge:

- a. To construct an angle congruent to a given angle.
- b. To construct the perpendicular bisector of a line segment.
- c. To construct a line segment congruent to a given segment.
- d. To construct a perpendicular to a line through a point not on the line.
- e. To bisect an angle.

(5) To verify constructions using congruent triangles.

(6) To recognize and use the reflexive, symmetric, and transitive properties.

(7) To use congruent triangles to prove segments or angles congruent.

Materials Needed

Set 2: compass, ruler, protractor

Set 4: scissors, packet I

Set 5: compass, straightedge

Set 7: half sheet of tracing paper

Suggestions for Teaching

Set 1, pages 2 - 5

Vocabulary: relation, congruence, congruent to

Symbols: ≅, ≇

Comments: Students should find this set relatively easy. The purpose of the set is to introduce them to an intuitive definition for congruent figures (e.g., same size and shape) and the problems with it.

DISCUSSION IS IMPORTANT

Have students discuss their answers to Exercises 6 through 23. You may want to ask questions like:

- -- Why are the figures congruent?
- -- How did you determine that the figures were congruent?

Be sure to discuss carefully with the students their answers to Exercises 24 and 25.

In discussing some of the exercises, arguments may arise as to what is the "correct" answer. For example, students who say that the figures in Exercise 8 on page 4 are *not* congruent since the size of one eye is larger are *correct*. Students who say that they assumed that the eyes are the same size (the error being the result of printing) and thus the figures *are* congruent are *also* correct, i.e., the conclusions should be dependent on the assumptions made by the student. Those who say the figures are *about* congruent should be pressed to decide one way or the other on the basis of their assumptions.

Set 2, pages 6 - 10

Vocabulary: line segment, congruent segments, angle, congruent angles

Symbols: $\overline{AB}$, ∠A, ∠EFG

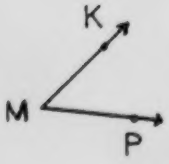
Materials Needed: compass, ruler, protractor

Comments: For Exercise 2 on page 6, students should first "eyeball" or estimate the congruent pairs of segments. Then have them check by using a ruler or a compass.

On page 7, students are told that a compass can be used to check distances. You may want to have students demonstrate this to the rest of the class.

Some of the exercises in Exercises 6 - 13 on page 8 are deliberately ambiguous. For example, the answer to Exercise 8 is "yes," if $CD = EF$ and "no," if $CD \neq EF$. Be sure to discuss these carefully!

It is assumed that students are already familiar with measuring angles in degrees, however, a short review is recommended. You may want to demonstrate the use of the protractor to measure angles A, B, C, and D on page 9 as well as to show the use of the protractor to draw angles. The demonstrations can be done on an overhead with a small transparent protractor.



Familiarize students with the notation used for angles. Show them an angle like the one on the left and ask them to suggest a notation for it. Most students are familiar with calling it "$\angle M$"; "$\angle KMP$" or "$\angle PMK$" are notations less known.

On the bottom of page 9, students are asked to examine congruent angles with different lengths of sides of the angles. Be sure to emphasize that the size of an angle does not depend on the length of its sides, but on how far apart its sides are. You may also want to point out that the "arrows" indicate that the rays "go on forever."

Students should realize that the answer to Exercise 22 on page 10 is true even if they know nothing about angle P.

Set 3, pages 11 - 15

Vocabulary: corresponding angles, corresponding segments

Symbols: $ABC \leftrightarrow FDE$, $\triangle LMN$

Comments: Have the students read and do the exercises in this set on their own. Then discuss their results as a class. Be sure to emphasize that the order in which the letters are written in a matching is important.

On page 13, the use of markings to show congruence is introduced. Spend some time discussing their use.

QUIZ 1

Anytime after Set 3

Set 4, pages 16 - 25

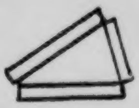
Vocabulary: included

Materials Needed: scissors, packet I

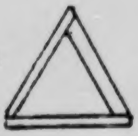
Comments: Students are asked to make models to verify the SAS, SSS, and ASA Postulates. Be sure to have students do the activities, instead of only talking about them. You may want to have students work in pairs for this set. Each pair of students should be provided with a packet of centimeter strips and a pair of scissors. Instruct students to cut the strips, if they need to.

Be sure to demonstrate how to correctly form the triangles.

This is correct:



NOT this way:



HELP STUDENTS READ!

As the students work, you should help them to read, reread, and follow the directions. A demonstration beforehand is not recommended.

Since the set is long, it is recommended that several class discussions be held during the study of the set. For example, students could come together to discuss their results after page 20, page 23, and page 25.

In the discussion after page 25, you may want to ask questions like:

---What parts of a pair of triangles must be congruent in order for the triangles to be congruent by SAS? SSS? ASA?

---How do SSS, SAS, and ASA compare?

---Could there be an AAA? What about SSA?

Have students explain their answers for the exercises on pages 24 and 25. (Transparency masters for these pages are provided.)

QUIZ 2 Anytime after Set 4

Set 5, pages 26 - 30

Vocabulary: construct, arc, perpendicular bisector, perpendicular, right angle, bisect, midpoint

Symbols: $\overrightarrow{EF}$, EF, $\overleftrightarrow{EF}$

Materials Needed: compass, straightedge

Comments: The following constructions are covered in this set:

1) To construct an angle congruent to a given angle.
2) To construct the perpendicular bisector of a line segment.
3) To construct a line segment congruent to a given line segment.
4) To construct a perpendicular to a line through a point on the line.
5) To bisect an angle.

You should work with the students to read, reread, and follow directions.

After they have had the opportunity to try to do the constructions by following the directions in the text, you may want to call on students to demonstrate the constructions step by step on the overhead or on the blackboard.

Check by Estimating!

Remind students to check their constructions by estimating. Oftentimes, they "go through the motions" without thinking about what they are doing.

For additional practice on constructions, assign supplementary worksheet 1.

Point out to the students that the prefix *"bi-"* means "two," as in bicycle, bimonthly, and bifocal. This may help them remember the meaning of the word, "bisect."

PROBLEM SOLVING: ALTERNATIVE METHODS

Discuss with the students the various ways to do Exercise 21 on page 31. Some students may explain that they just made a copy of the perpendicular bisector of $\overline{AD}$ (from Exercise 19). Other students may explain that they made a copy of $\overline{MN}$ and added it to their half of $\overline{MN}$.

On page 32, students are asked to construct a line perpendicular to a given line through a given point not on the line. You may want to help the students with this problem by asking questions such as:

UNDERSTANDING THE PROBLEM

---What is the problem?
---What does "perpendicular" mean?

DEVISING A PLAN

---Have you solved a problem similar to this?
---How can you use the hint--use your compass to find two points, B and C, on L so that A would be the same distance from B as from C?

CARRYING OUT

---Can you demonstrate each step?

LOOKING BACK

---Have you solved the problem?

Have students find a line perpendicular to a given line through a point on the line. (optional)

QUIZ 2

Anytime after Set 5

Set 6, pages 35 - 37

Vocabulary: symmetry, symmetric, reflexivity, reflexive, transitivity, transitive

Comments: Have students explain the reasons for their answers to the exercises in this set.

Stress the properties for the relations and not instances of the relations.

Set 7, pages 38 - 42

Vocabulary: vertical angles, linear pair, supplementary angle, supplement, isosceles triangle, equilateral triangle, equiangular triangle

Symbols: $m \angle HJF$

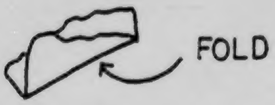
Materials Needed: Half sheet of tracing paper

Comments: Many new terms are introduced in this set. Focus on providing practice for the students in identifying the "vertical angles," "supplementary angles," "isosceles triangles," and so on, in drawings as well as practice in drawing the angles and triangles themselves.

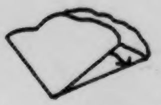
You may have to help students with the directions on page 39 to show that "vertical angles are congruent."

You may want to have students do the following activity on perpendicularity:

(1) Take any sheet of paper and fold it.



(2) Now fold again bringing the first folded edges together.



(3) Unfold the sheet. Can you explain why the fold lines are perpendicular to each other?

Set 8, pages 43 - 44

Comments: Prior to the introduction of formal proofs, students must develop skill in drawing valid conclusions from given information. To answer the exercises in this set, students must use conclusions made in the previous sets. Initially some of the students may have difficulty giving explanations for the exercises. Remind them that although the pictures may suggest the answers, they should include in their explanations some of the ideas about congruence that were learned in Sets 1 - 7.

PROBLEM - SOLVING STRATEGIES

It is recommended that you assign a few exercises the first time, have students discuss their answers as a class, and then assign the rest of the exercises. During the discussion you should ask questions such as:

---What information were you given?
---What is the problem?
---What is the answer?
---How did you get the answer?
---Did anyone get their answer another way?

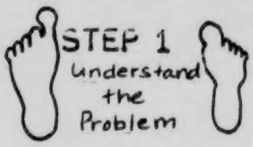
Set 9, pages 45 - 53

Vocabulary: definitions, postulates, theorems, adjacent

Comments: Formal definitions, postulates, and theorems for Unit I are presented in this set. Students may want to make a few additions and/or corrections to their lists. For example, "adjacent angles" are introduced in this set.

Read through the set with the students. Review each definition, postulate, and theorem carefully with them.

The proofs in this text are intended primarily to expose students to and acquaint them with correct mathematical language and presentation rather than "mastery" and proficiency in constructing proofs themselves. Read and discuss the proofs with the students. The four steps to problem solving and problem-solving strategies should be emphasized. The first step is to understand the problem. You may want to ask the following questions:



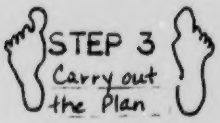
---Can you restate the problem?
---Do you know the meaning of the terms used in the problem?
---Can you draw a picture (if one is not provided)?
---Can you list all the information given and the related facts?

The next step is to devise a plan for the proof. It is important for the students to get a sense of the logical chain of the argument before actually writing the proof. To help them with this step, you may want to ask the following questions:

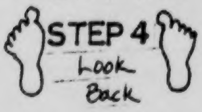
STEP 2
Devise a Plan

---Have you solved a problem like this before?
---Can you work the problem backwards?
---Can you list possible alternatives?

Help students present oral arguments for their proofs.



The third step is to help students present written proofs from their oral arguments.

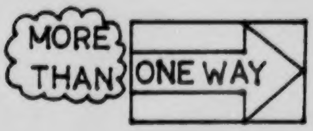


The last step is very important. Be sure to have students look back at their work. Some possible questions to ask include:

---Was the plan a good one?
---Have you proven what was asked for?

NOTE: The proofs shown in this textbook are all paragraph proofs. Paragraph proofs are recommended.

Be sure to distinguish between an outline of a proof and a proof--a proof is written in complete sentences!



Be sure to emphasize that there may be more than one way to do a proof. Oftentimes, students get frustrated because they feel their proofs are different from the "correct" proof.

? FORMAL PROOF ? ! ARGUMENTS FOR CONCLUSIONS !

Try not to be overly fussy about details in the students' proofs. With more experience, many students will appreciate the finer points of logical arguments.

DO NOT require that every student be able to do a formal proof. Every student, however, should be able to present arguments for conclusions.

Discuss the proofs for Theorems 1 and 2 as a class. Then do Theorems 3, 4, and 5 as a class. Remind students about the four steps to problem solving.

Assign a few proofs at a time from pages 51 - 53. Have some students write their proofs on the blackboard and discuss them as a class.

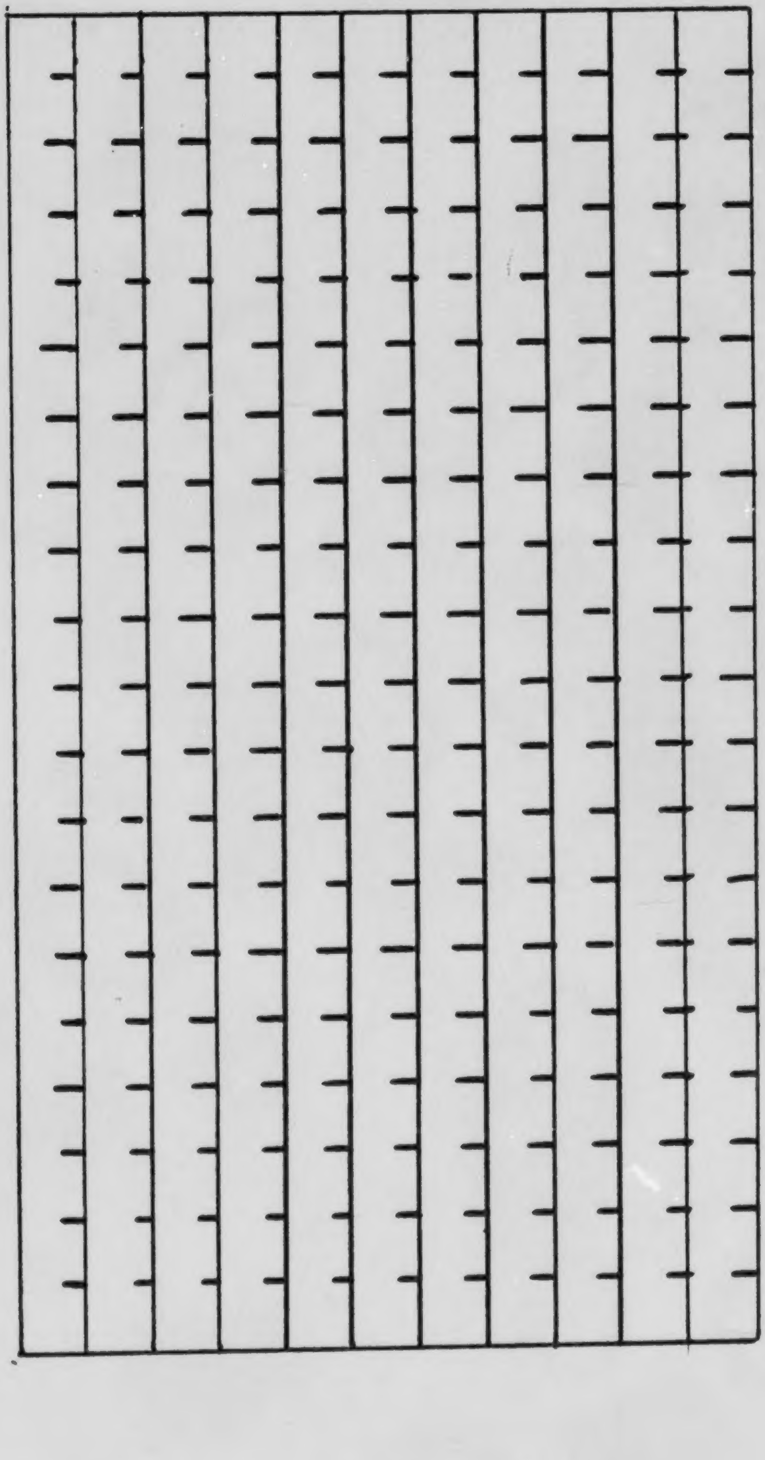
USE MODELS To help students "see" the figure in Exercise 7, you may want to use cardboard models.

QUIZ 4 During the study of Set 9

Review

A review worksheet is provided.

UNIT I TEST



Directions: 1) Make a ditto master of this sheet.

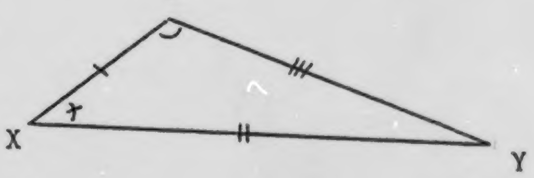
2) Ditto copies on 5" by 8" index cards, if available.

3) Cut the centimeter strips lengthwise and place them into envelopes.

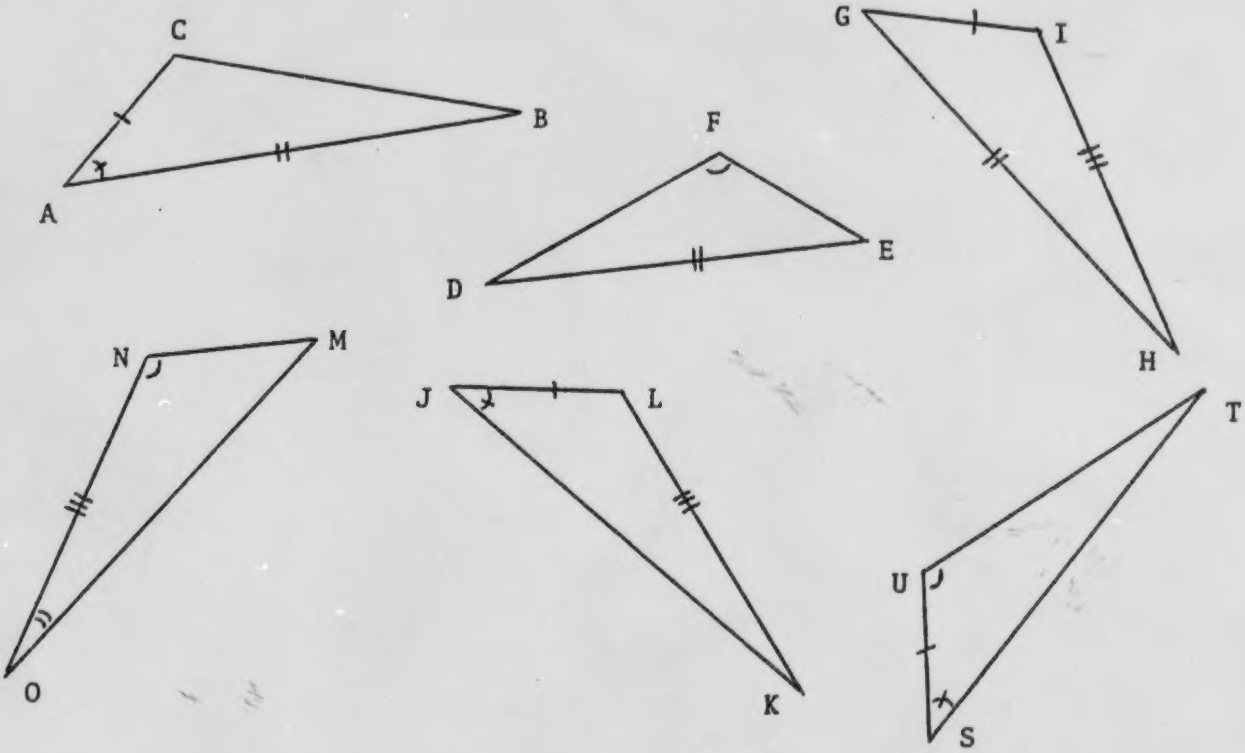
Transparency 1

Unit I Set 4

Here is ΔXYZ:



From the pictures below, choose all the triangles which are congruent to ΔXYZ, and tell which of SAS, SSS, or ASA applies. (Remember that once two triangles are congruent, all the corresponding parts are congruent.)

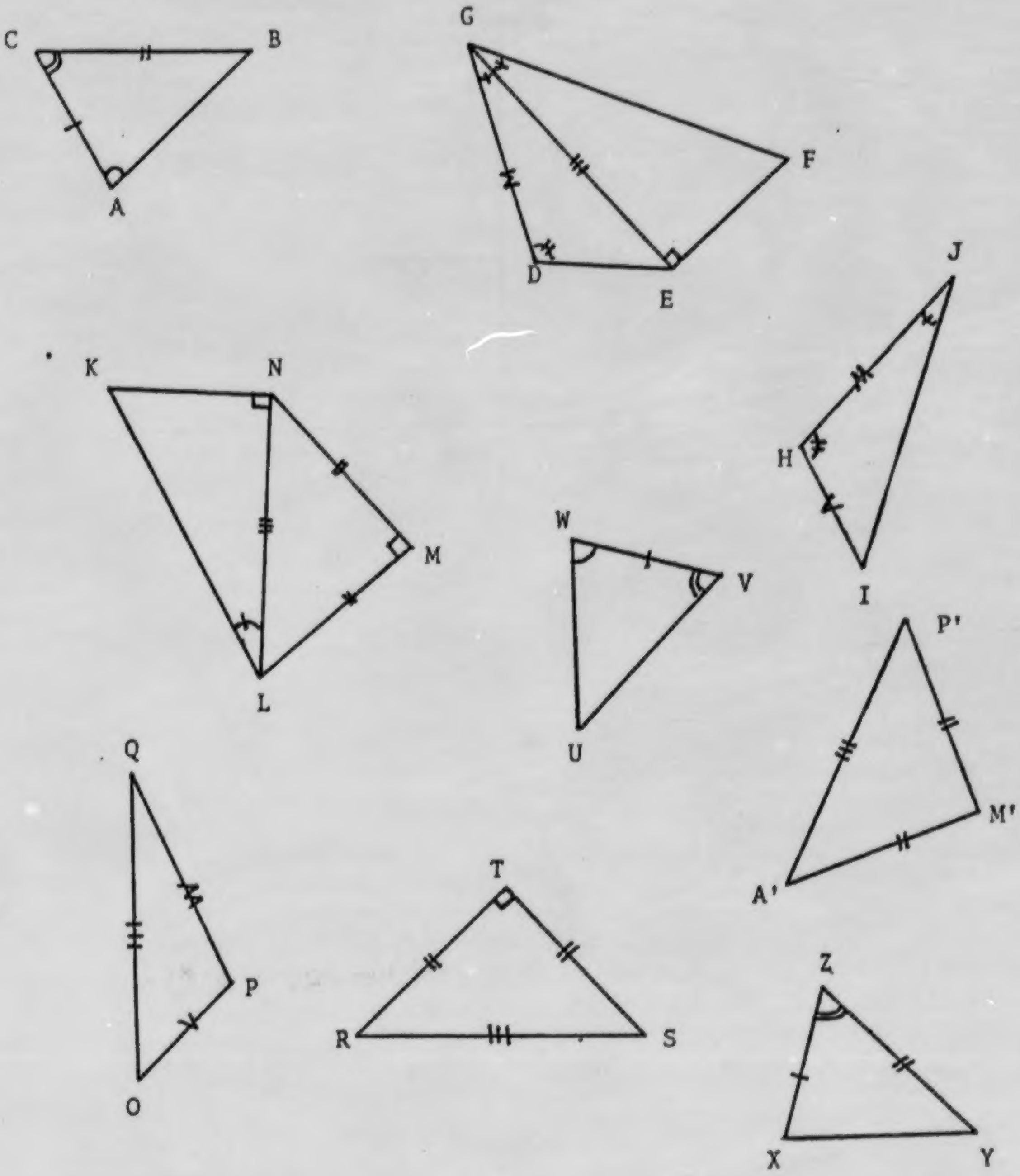


41. What additional information would you want before you could say that ΔJKL is congruent to ΔABC?

From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 24.

Transparency 2
Unit I   Set 4

Shown below are triangles with congruent parts designated. Choose the triangles which are congruent, applying SAS, SSS, or ASA.


C
B
A
G
F
D
E
J
K
N
H
M
W
V
I
L
U
P'
Q
M'
T
A'
P
R
S
Z
O
X
Y

From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 25.

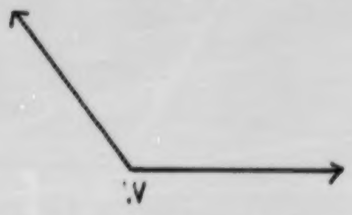
Unit I, Set 5
Supplementary Worksheet 1

Name ______________________

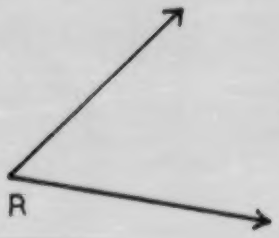
Use only a compass and a straightedge for all constructions!
Be sure to leave the compass markings.

1. Construct the angle bisectors of the angles below:

a.

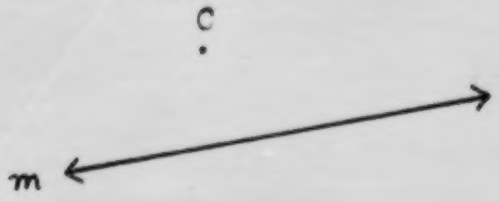


b.

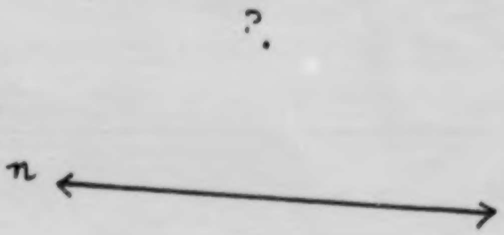


2. Construct a line perpendicular to each given line through the point not on the line.

a.



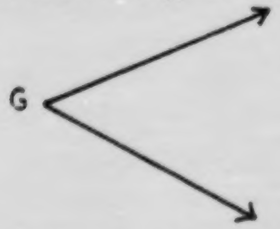
b.



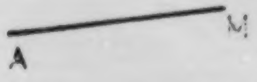
DO THE FOLLOWING CONSTRUCTIONS ON ANOTHER SHEET OF PAPER.

3. Construct a copy of the angle and the line segment below:

a.

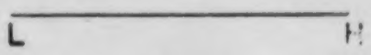


b.



4. Construct a line segment that is twice as long as the segment in Exercise 3b above.

5. Draw or construct a copy of the line segment below. Then construct the perpendicular bisector of the line segment.



Unit I
Quiz 1

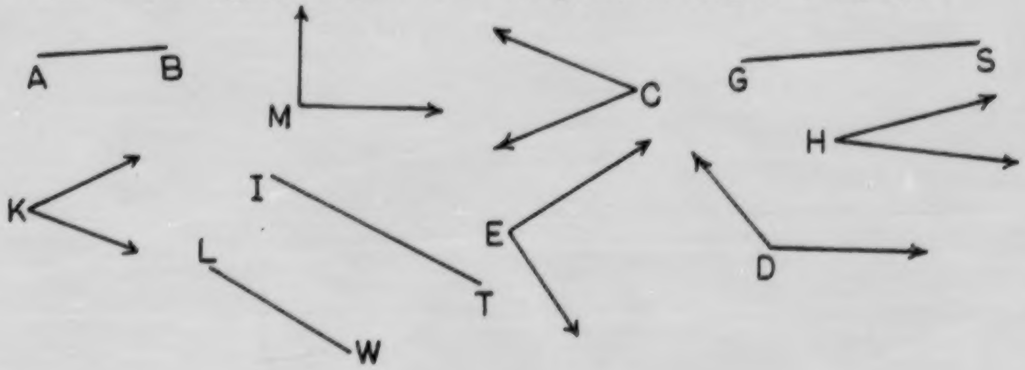
Name ______________________________

1. When are two figures congruent?

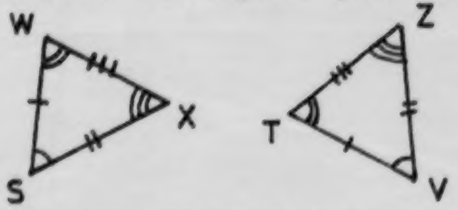
2. When are two line segments congruent?

3. When are two angles congruent?

4. Look at the figures below. Name the pairs of congruent figures.



5. Given Δ SWX and Δ TVZ with congruent parts shown in the figures below.



Check each of the following matchings to see whether they show that the triangles are congruent.

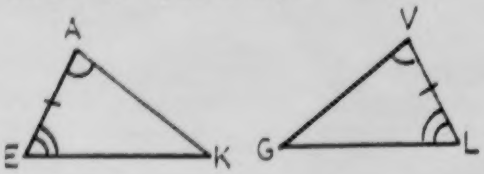
_____	a. SWX ⟷ TVZ	_____	c. SWX ⟷ VTZ
_____	b. SWX ⟷ TZV	_____	d. SWX ⟷ ZTV

Unit I
Quiz 2

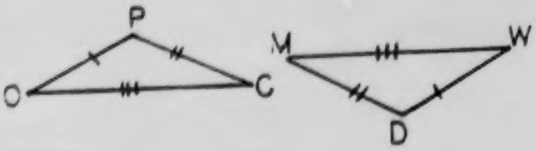
Name ______________________________

Examine each pair of triangles below and decide if they are congruent given the congruent parts marked. Write "no,"if the triangles are not congruent. If they are congruent, name the triangles which are congruent and state the reason why.

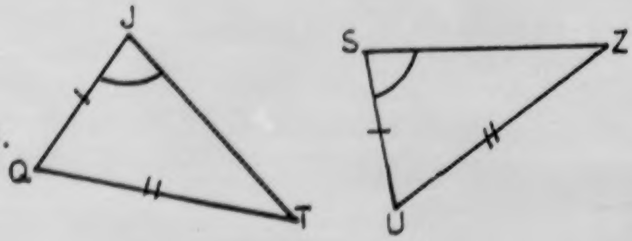
1.



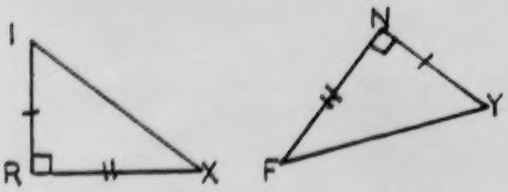
2.



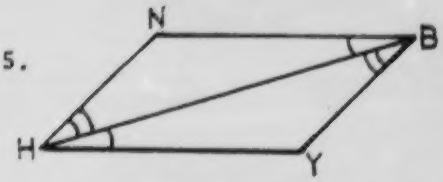
3.



4.



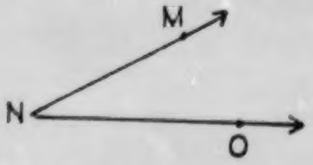
5.

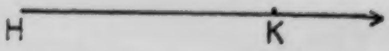


Unit I
Quiz 3

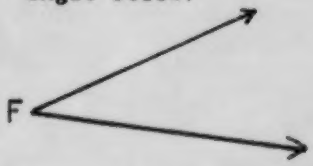
Name ______________________

1. Using a compass and a straightedge, construct an angle congruent to ∠ MNO with $\overline{HK}$ as one side.

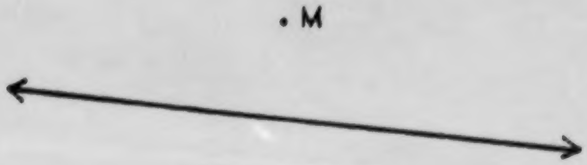




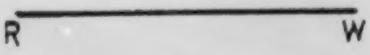
2. Using a compass and a straightedge, construct the bisector of the angle below.



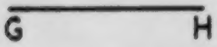
3. Using a compass and a straightedge, construct a line perpendicular to the line below through the point M.



4. Using a compass and a straightedge, construct the perpendicular bisector of $\overline{RW}$.



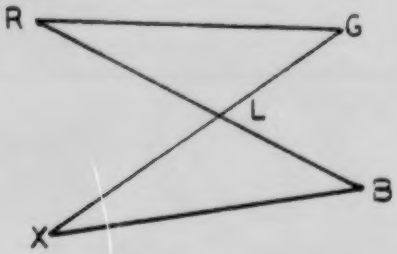
5. Using a compass and a straightedge, construct a line segment that is twice as long as segment GH. Label the enlarged segment, $\overline{TV}$.



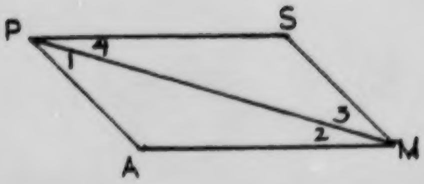
Unit I
Quiz 4

Name ______________________

1. Suppose $\overline{RL} \cong \overline{XL}$ and $\angle R \cong \angle X$. What can you say about $\triangle RLG$ and $\triangle XLB$? Explain.



2. If $\overline{AP} \cong \overline{SM}$ and $\angle 1 \cong \angle 3$, prove that $\overline{AM} \cong \overline{SP}$.

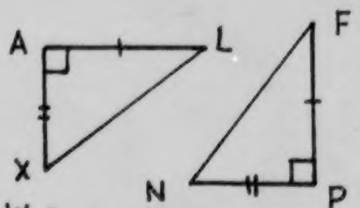


Unit I
Review Worksheet

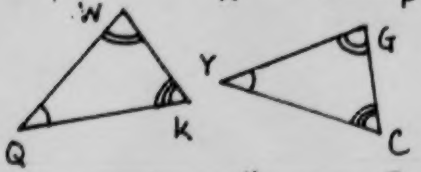
Name ______________________________

1. Examine each pair of triangles below and decide if they are congruent given the congruent parts marked. Write "no" if the triangles are not congruent. If they are congruent, name the triangles which are congruent and state the reason why.

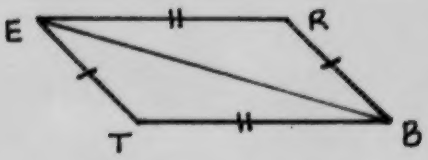
________ a.



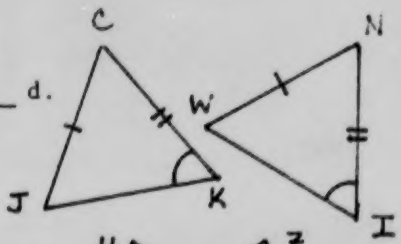
________ b.



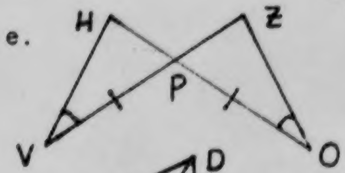
________ c.



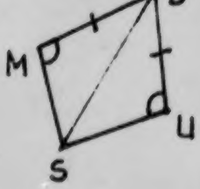
________ d.



________ e.

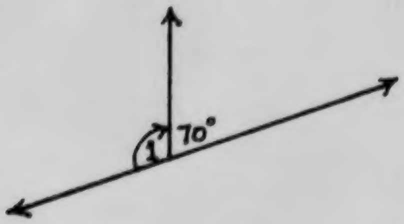


________ f.

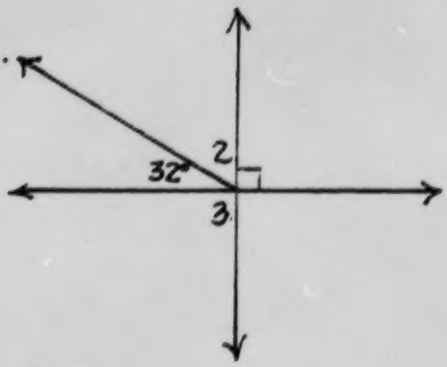


2. Find the measures of angles 1 - 8.

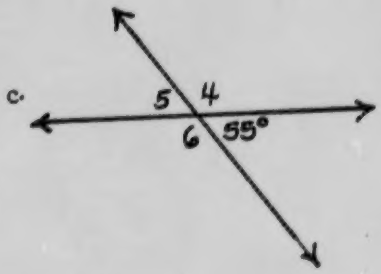
________ a.



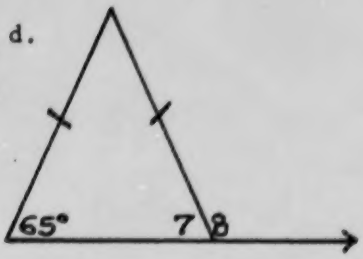
________ b.



____
____
________ c.



____
________ d.



(Go on to the next page.)

3. True or False: use + for true; 0 for false.

______ a. Every equiangular triangle is equilateral.

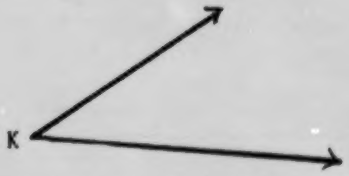
______ b. Two supplementary angles always form a linear pair.

______ c. If $\Delta BTU \cong \Delta MPG$, then $\angle T \cong \angle G$.

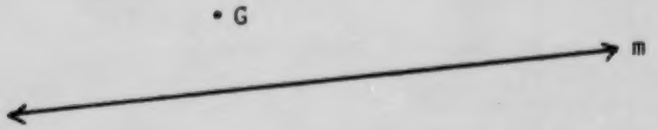
______ d. If $\angle K$ and $\angle W$ are vertical angles and $m\angle K = 80$, then $m\angle W = 100$.

______ e. If a line divides a line segment into two congruent parts, then the line bisects the line segment.

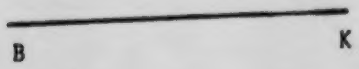
4. Using a compass and a straightedge, bisect angle K below:



5. Using a compass and a straightedge, construct a line perpendicular to line m through point G:



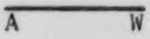
6. Using a compass and a straightedge, construct the perpendicular bisector of $\overline{BK}$.



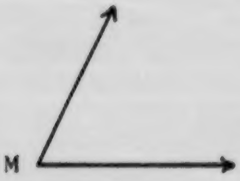
(Go on to the next page.)

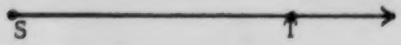
Name____________________

7. Using a compass and a straightedge, construct a line segment so that it is $4\frac{1}{2}$ times as long as $\overline{AW}$.



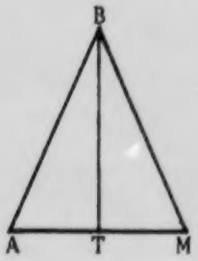
8. Using a compass and straightedge, construct a copy of angle M on $\overline{ST}$.



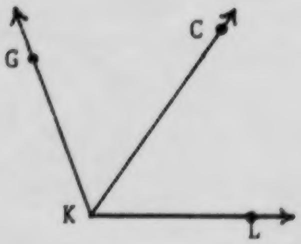


9. For numbers, which of the properties (reflexivity, symmetry, transitivity) does the relation "is one-third" have? Explain.

10. Suppose $\overline{BT}$ is the perpendicular bisector of $\overline{AM}$ as shown. What conclusions can you make? (There are several conclusions that can be made.)



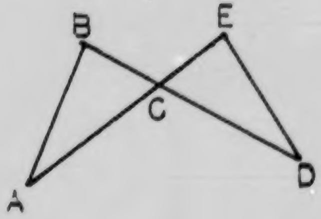
11. Suppose $\overrightarrow{KC}$ bisects angle GKL. What conclusions can you make?



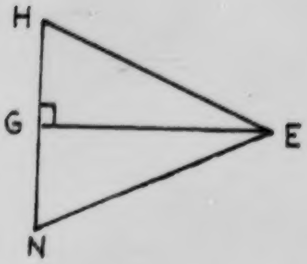
(Go on to the next page.)

Name ___________________________

12. Suppose $\overline{AC} \cong \overline{DC}$ and $\overline{CE} \cong \overline{CB}$. What can you say about $\angle A$ and $\angle D$? Explain why.

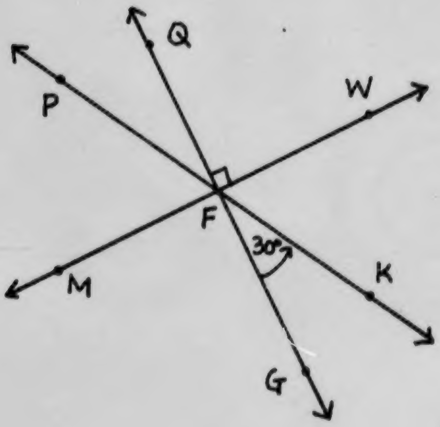


13. Suppose $\overline{EG}$ is perpendicular to $\overline{HN}$ and $\overline{EG}$ bisects $\angle$ HEN, prove that Δ EHG $\cong$ Δ ENG.



Unit I Test Name ________________________

Use the figure on the right to answer questions 1 - 5.



________ 1. Name a linear pair of angles.

________ 2. Name a pair of vertical angles.

________ 3. Name a line perpendicular to $\overleftrightarrow{GQ}$.

________ 4. ∠ PFM and __?__ are adjacent angles.

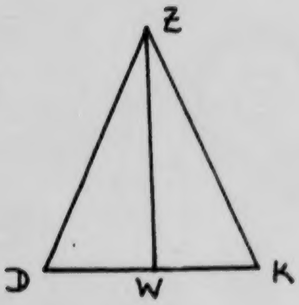
5. Find the following angle measures:

________ a. m ∠ QFK

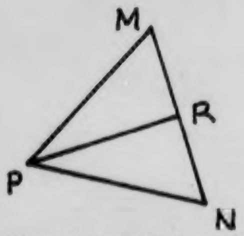
________ b. m ∠ PFW

________ c. m ∠ MFG

6. If $\overline{ZW}$ is the perpendicular bisector of $\overline{DK}$, what kind of triangle is △ DKZ? Why?



7. If $\overline{PR}$ bisects ∠ P and $\overline{PM} \cong \overline{PN}$, why is △ PMR ≅ △ PNR?

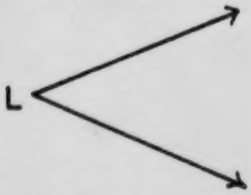


8. For numbers, which of the properties (reflexivity, symmetry, transitivity) does the relation "is greater than" have? Explain.

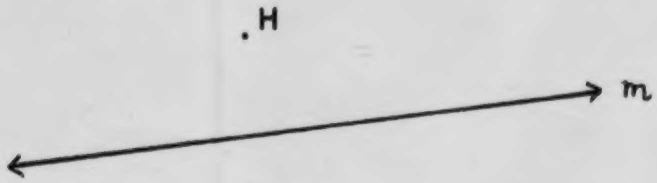
(Go on to the next page.)

9. Make the following constructions. Be sure to use ONLY a compass and a straightedge!

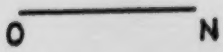
   a. Construct the bisector of angle L below:



   b. Construct a line perpendicular to line m that goes through point H.



   c. Construct a line segment that is $2\frac{1}{2}$ times as long as $\overline{ON}$ below:



10. Suppose each of two triangles has a right angle and side lengths of 4 cm and 6 cm. Do you have enough information to know whether the triangles are congruent or not? Explain.

11. Tell whether each of the following statements is true or false.

_______ a. If $\triangle BEV \cong \triangle TWY$, then $\overline{EV} \cong \overline{TY}$.

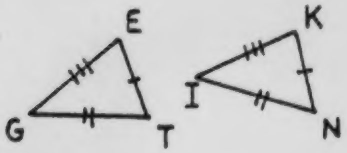
_______ b. If a triangle is equilateral, then it must be isosceles.

_______ c. If $\angle A$ and $\angle B$ are supplementary angles and $m \angle A = 75$, then $m \angle B = 15$.

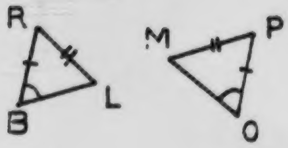
(Go on to the next page.)

12. Examine each pair of triangles below and decide if they are congruent given the congruent parts marked. Write "no" if the triangles are not congruent. If they are congruent, name the triangles which are congruent and state the reason why.

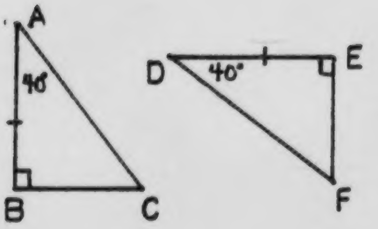
a.



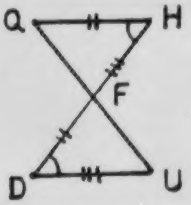
b.



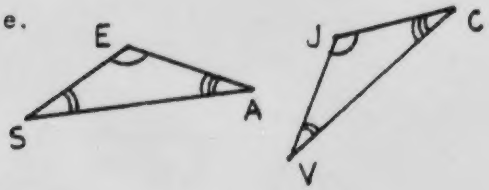
c.


A
40°
D
40°
E
B
C
F

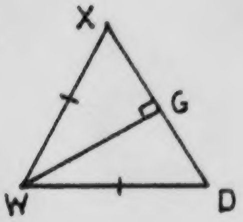
d.



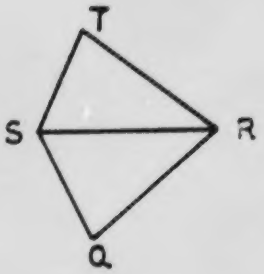
e.



f.



13. Suppose $\overline{RT} \cong \overline{RQ}$ and $\angle TRS \cong \angle QRS$, prove that $\angle TSR \cong \angle QSR$.

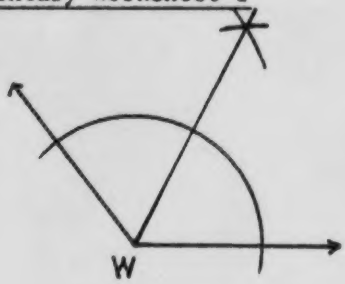


ANSWER KEY

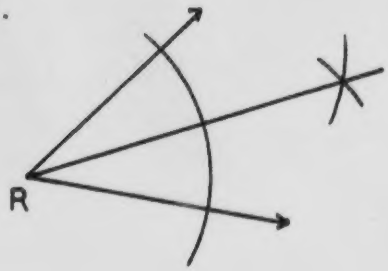
NOTE: There may be more than one answer for some of the questions.

Supplementary Worksheet 1

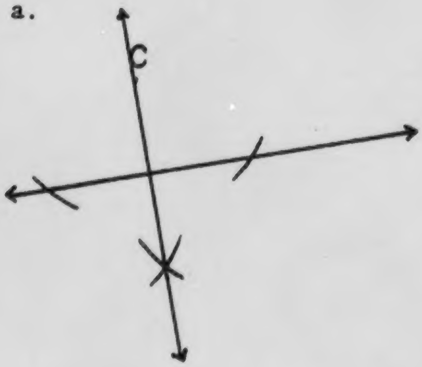
1. a.



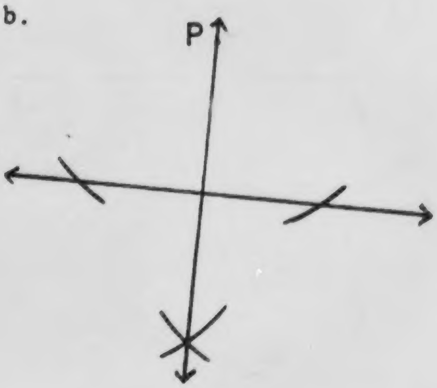
b.



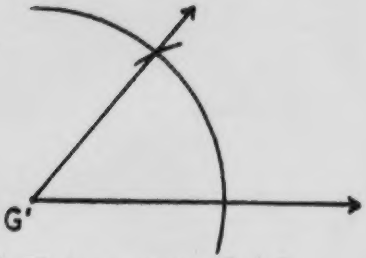
2. a.



b.

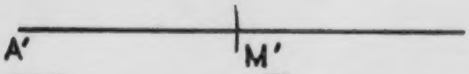


3. a.



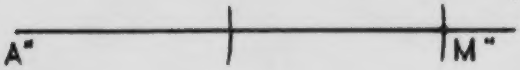
$\angle G'$ is a copy of $\angle G$.

b.



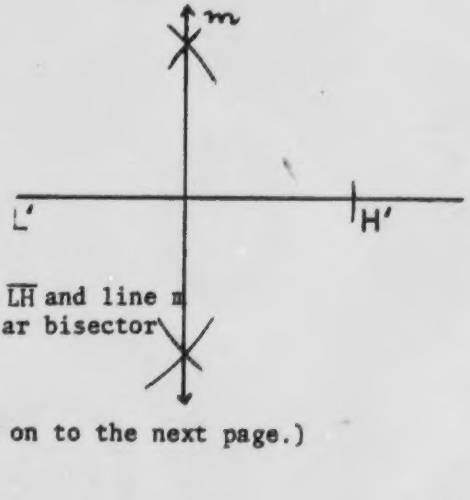
$\overline{A'M'}$ is a copy of $\overline{AM}$.

4.



$\overline{A''M''}$ is twice as long as $\overline{AM}$.

5.



$\overline{L'H'}$ is a copy of $\overline{LH}$ and line m is the perpendicular bisector of $\overline{L'H'}$.

(Go on to the next page.)

Quiz 1 (*10 points*)

Answers may vary for Questions 1 - 3. Here are possible answers:

(1) 1. Two figures are congruent if they are the same size and shape.

(1) 2. Two line segments are congruent if and only if they are the same length.

(1) 3. Two angles are congruent if and only if they have the same measure.

(1) 4. $\angle M \cong \angle E$, $\angle C \cong \angle K$, $\overline{GS} \cong \overline{IT}$

(1) 5. a. No b. No c. Yes d. No

Quiz 2 (*10 points*)

(2) 1. $\triangle$ AEK $\cong$ $\triangle$ VLG by ASA Congruence

(2) 2. $\triangle$ COP $\cong$ $\triangle$ MWD by SSS Congruence

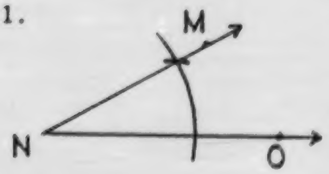
(2) 3. No

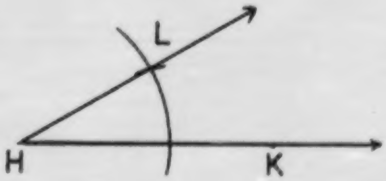
(2) 4. $\triangle$ IRX $\cong$ $\triangle$ YNF by SAS Congruence

(2) 5. $\triangle$ BHY $\cong$ $\triangle$ HBN by ASA Congruence

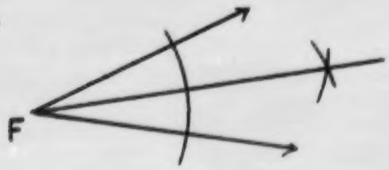
Quiz 3 (*10 points*)

(2) 1.

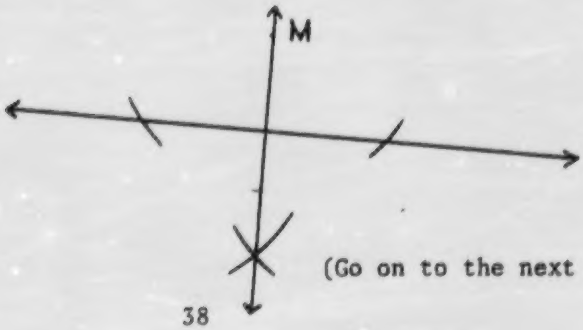




(2) 2.

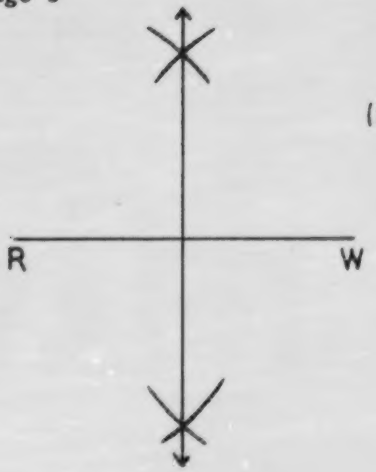


(2) 3.

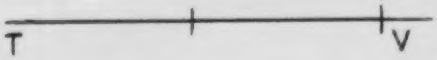


(Go on to the next page.)

(2) 4.



(2) 5.



Quiz 4 *(10 points)*

Answers may vary. Here are possible answers:

(5) 1. $\triangle RLG \cong \triangle XLB$. $\angle RLG \cong \angle XLB$ since they are vertical angles. Since $\overline{RL} \cong \overline{XL}$, $\angle R \cong \angle X$, and $\angle RLG \cong \angle XLB$, it follows that $\triangle RLG \cong XLB$ by ASA.

(5) 2. Suppose $\overline{AP} \cong \overline{SM}$ and $\angle 1 \cong \angle 3$. By the reflexivity property $\overline{PM} \cong \overline{PM}$. Then $\triangle AMP \cong \triangle SPM$ by SAS. Therefore, $\overline{AM} \cong \overline{SP}$ by the definition of congruent triangles.

Unit I Review Worksheet

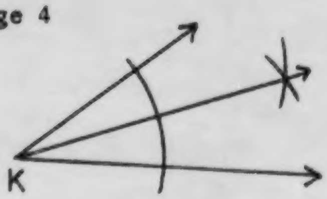
1. a. $\triangle ALX \cong \triangle PFN$ by SAS Congruence
   b. No
   c. $\triangle BET \cong \triangle EBR$ by SSS Congruence
   d. No
   e. $\triangle HPV \cong \triangle ZPO$ by ASA Congruence
   f. No

2. a. $m\angle 1 = 110$
   b. $m\angle 2 = 58$
      $m\angle 3 = 90$
   c. $m\angle 4 = 125$
      $m\angle 5 = 55$
      $m\angle 6 = 125$
   d. $m\angle 7 = 65$
      $m\angle 8 = 115$

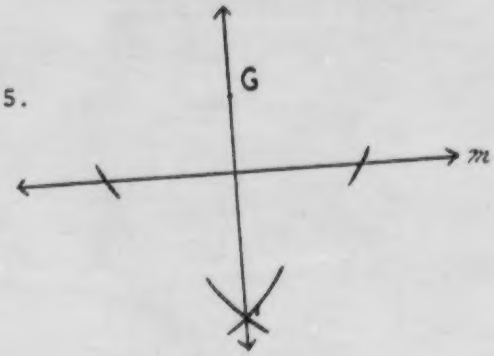
3. a. + b. 0 c. 0 d. 0 e. +

(Go on to the next page.)

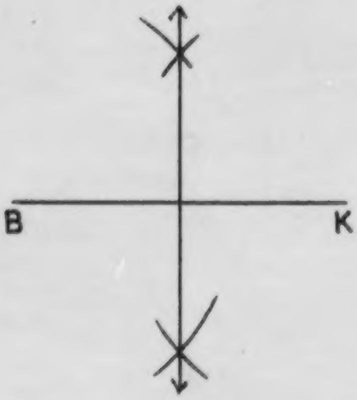
4.



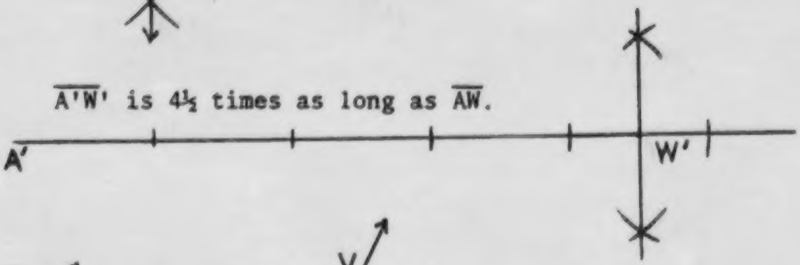
5.



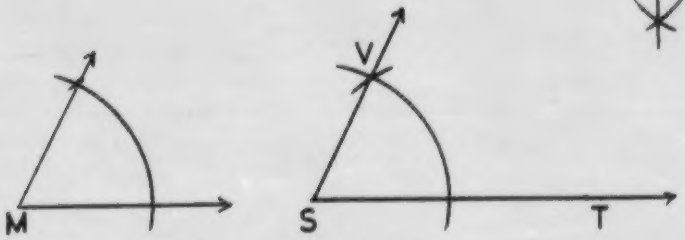
6.



7. $\overline{A'W'}$ is $4\frac{1}{2}$ times as long as $\overline{AW}$.



8.



∠VST is a copy of ∠M.

9. The relation "is one-third of" for numbers does not have any of the properties. It is not reflexive since, for example, 5 is not one-third of 5. It is not symmetric since, for example, 5 is one-third of 15, but 15 is not one-third of 5. It is not transitive since, for example, 5 is one-third of 15 and 15 is one-third of 45, but 5 is not one-third of 45.

10. Answers will vary. Here are some possible answers:

1) ∠ATB and ∠BTM are right angles

2) ∠ATB ≅ ∠BTM

3) $\overline{AT} \cong \overline{TM}$

4) $\triangle ABT \cong \triangle MBT$

5) $\overline{AB} \cong \overline{BM}$

6) ∠A ≅ ∠M

7) ∠ABT ≅ ∠MBT

(Go on to the next page.)

11. $\angle GKC \cong \angle LKC$

12. Answers may vary. Here is a possible answer:

$\angle A \cong \angle D$. Suppose $\overline{AC} \cong \overline{DC}$ and $\overline{CE} \cong \overline{CB}$. Since $\angle ACB$ and $\angle DCE$ are vertical angles, $\angle ACB \cong \angle DCE$ by Theorem 1. It follows that $\triangle ABC \cong \triangle DEC$ by SAS Congruence. Therefore, $\angle A \cong \angle D$ by the definition of congruent triangles.

13. Answers may vary. Here is a possible answer:

Suppose $\overline{EG}$ is perpendicular to $\overline{HN}$ and $\overline{EG}$ bisects $\angle HEN$. By the definition of perpendicular line segments, $\angle EGH$ and $\angle EGN$ are right angles. Then $\angle EGH \cong \angle EGN$. $\angle GEH \cong \angle GEN$ by the definition of an angle bisector and $\overline{EG} \cong \overline{EG}$ by the reflexive property. Therefore, $\triangle EHG \cong \triangle ENG$ by ASA.

Unit I Test *(50 points)*

(1) 1. Answers may vary. Here are a few possible answers:

(1) $\angle MFP$ and $\angle PFW$
(2) $\angle MFQ$ and $\angle QFW$
(3) $\angle PFQ$ and $\angle KFQ$
(4) $\angle GFM$ and $\angle GFW$

(1) 2. Answers may vary. Here are a few possible answers:

(1) $\angle MFP$ and $\angle KFW$
(2) $\angle GFM$ and $\angle QFW$
(3) $\angle GFK$ and $\angle PFQ$

(1) 3. $\overleftrightarrow{MW}$

(1) 4. Answers may vary. Here are some possible answers:

(1) $\angle PFQ$
(2) $\angle PFW$
(3) $\angle PFK$
(4) $\angle KFM$
(5) $\angle GFM$

(6) 5. a. $m\angle QFK = 150$ b. $m\angle PFW = 120$ c. $m\angle MFG = 90$

(4) 6. It is an isosceles triangle. Since $\overline{ZW}$ is the perpendicular bisector of $\overline{DK}$, then $\overline{DZ} \cong \overline{KZ}$. By the definition of an isosceles triangle, $\triangle DKZ$ is an isosceles triangle.

OR

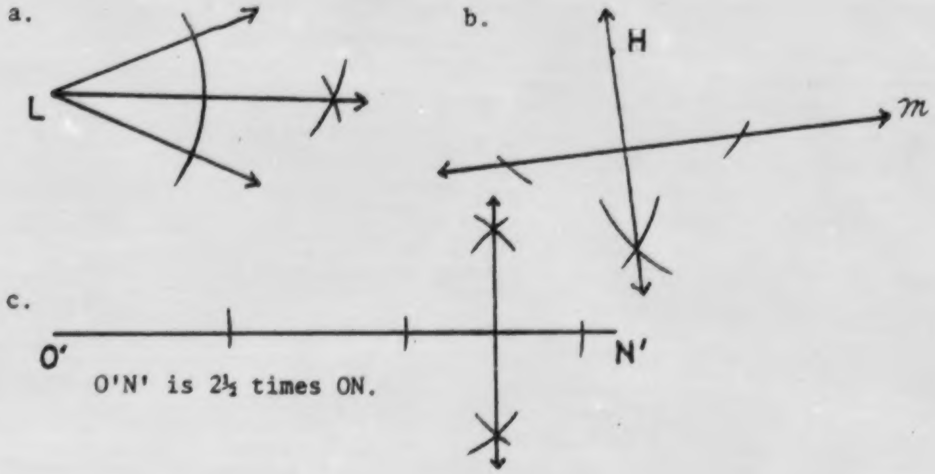
Since $\overline{ZW}$ is the perpendicular bisector of $\overline{DK}$, it follows by the definition of a perpendicular bisector that $\overline{DW} \cong \overline{KW}$ and $\angle DWZ$ and $\angle KWZ$ are 90-degree angles. Then $\angle DWZ \cong \angle KWZ$. Since $\overline{DW} \cong \overline{KW}$, $\angle DWZ \cong \angle KWZ$, and $\overline{ZW} \cong \overline{ZW}$ (by the reflexivity property), then by SAS, $\triangle DWZ \cong \triangle KWZ$. Therefore, $\overline{DZ} \cong \overline{KZ}$. By the definition of an isosceles triangle, $\triangle DKZ$ is an isosceles triangle.

(Go on to the next page.)

(4) 7. Since $\overline{PR}$ bisects $\angle$ P, $\angle$ MPR $\cong$ $\angle$ NPR. From $\angle$ MPR $\cong$ $\angle$ NPR, $\overline{PM} \cong \overline{PN}$, and $\overline{PR} \cong \overline{PR}$ (by the reflexivity property), it follows that Δ PMR $\cong$ Δ PNR by SAS.

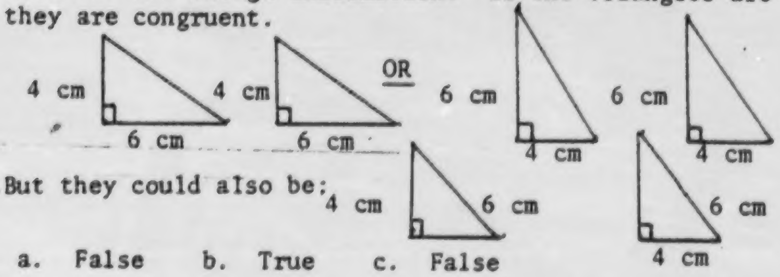
(3) 8. The relation "is greater than" for numbers has the transitivity property. For example, if $8 > 4$ and $4 > 1$, then $8 > 1$. It is not reflexive since, for example, $2 \not> 2$. It is also not symmetric since, for example, $5 > 2$, but $2 \not> 5$.

(6) 9.



O'N' is 2½ times ON.

10. There is not enough information. If the triangles are as below, they are congruent.



But they could also be:

11. a. False b. True c. False

12. a. Δ EGT $\cong$ Δ KIN
by SSS Congruence
b. No
c. Δ ABC $\cong$ Δ DEF
by ASA Congruence
d. Δ FHQ $\cong$ Δ FDU
by SAS Congruence
e. No
f. No

13. Answers may vary. Here is a possible answer:

Suppose $\overline{RT} \cong \overline{RQ}$ and $\angle$ TRS $\cong$ $\angle$ QRS. $\overline{RS} \cong \overline{RS}$ by the reflexivity property. Then Δ RST $\cong$ Δ RSQ by SAS. By the definition of congruent triangles, $\angle$ TSR $\cong$ $\angle$ QSR.

Unit II: Inequalities

Objectives

(1) To define and use betweenness of points.

(2) To define and use addition of measures for adjacent angles.

(3) To discover and apply the relationship that the measure of an exterior angle of a triangle is larger than each of its non-adjacent interior angles.

(4) To discover and apply the following inequality relations to a triangle:

  (a) If two sides of a triangle are not congruent, the angle opposite the longer side is larger.

  (b) If two angles of a triangle are not congruent, the side opposite the larger angle is longer.

  (c) The shortest segment from a point not on a line to the line is the perpendicular segment.

  (d) The sum of the lengths of two sides of a triangle is greater than the length of the third side.

Materials Needed

Sets 3 & 4: ruler

Set 5: packet II

Suggestions for Teaching

Set 1, pages 56 - 60

Vocabulary: collinear, between

Comments: Students should find this set very easy.

SEVERAL POSSIBLE ANSWERS

For Exercise 9 on page 57, students should be aware that there are several possibilities for placing the points on line M.

Be sure to have students discuss their answers for Exercise 21a and 21c on page 58. They should realize that there are several possible answers for both of these exercises.

Inform students that unless otherwise specified, betweenness and collinearity of points may be assumed if they are given in a diagram.

Set 2, pages 61 - 62

Vocabulary: exterior angle

Comments:

STUDY
DATA

LOOK
for
PATTERNS

In Exercise 8 on page 62, students are asked to study data collected in previous exercises in order to write a statement relating the exterior angle with the interior angles of a triangle. Have students present and explain their statements. Students should realize that the measure of an exterior angle of a triangle is larger than each of its nonadjacent interior angles. Some students may also notice that the measure of an exterior angle of a triangle is equal to the sum of the measures of its two nonadjacent interior angles. Although this second relationship is not stressed in the text, you may want to discuss it in class.

Set 3, pages 63 - 66

Vocabulary: converse

Comments:

STUDY
TABLES

Students are asked to study the information they have recorded in tables to see if there is any relationship between the sides and angles that are compared. Have students present their findings and discuss how they obtained them.

The term "converse" is introduced in Exercise 7 on page 64. Students should learn the difference between a statement and its converse.

It is recommended that the students discuss their answers to Exercises 1 through 7 as a class before trying to do Exercises 8 through 11.

QUIZ I

Anytime after Set 3

Set 4, page 67

Comments:

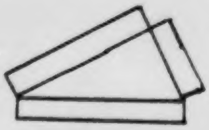
Exercises 5 and 6 could be used as discussion questions rather than problems for individual assignment:

#5. Choose a point R such that E is between D and R with R on the edge of the page. Can you find a point on L which would be farther from P than R? Can you find a point farther than that point? What about a point 1,000,000 km away? Is there another point farther away?

Set 5, pages 68 - 70

Materials Needed: Packet II

Comments: Have students work in pairs to answer the exercises on pages 68 and 69. Students should be cautioned to form the triangles like the one below:



Have some students enter their data into a table on the blackboard (like the one on page 69). Discuss the results as a class.

Set 6, pages 71 - 72

Comments: Students are asked to use conclusions from the previous set to do these exercises. It is recommended that a few exercises be assigned each time. Have students explain how they obtained their answers for these exercises before assigning the next set of exercises.

REVIEW INEQUALITY PROPERTIES

Exercises 7 and 8 on page 72 are difficult problems for most students. They assume that students are familiar with the inequality properties. A review of these properties may be necessary before students attempt these exercises.

QUIZ 2

Anytime after Set 6

Set 7, pages 73 - 82

Comments: Formal definitions, postulates, and theorems for Unit II are presented in this set. Allow time for students to study and discuss these carefully. They should be reminded to draw pictures as they read Postulates 6 and 7. Have them continue to keep their list of the definitions, postulates, and theorems.

PROBLEM SOLVING AND PROOFS

Since the proofs are intended primarily to expose students to and acquaint them with correct mathematical language and presentation rather than "mastery" and proficiency in constructing proofs themselves, the proofs should be reviewed with students. Be sure to discuss with the students the use of the four steps to problem solving (shown on the next page) when doing written proofs.

1) Understanding the problem
2) Devising a plan to solve the problem
3) Carrying out the plan
4) Looking back: reviewing the problem and solution

Take-home Test

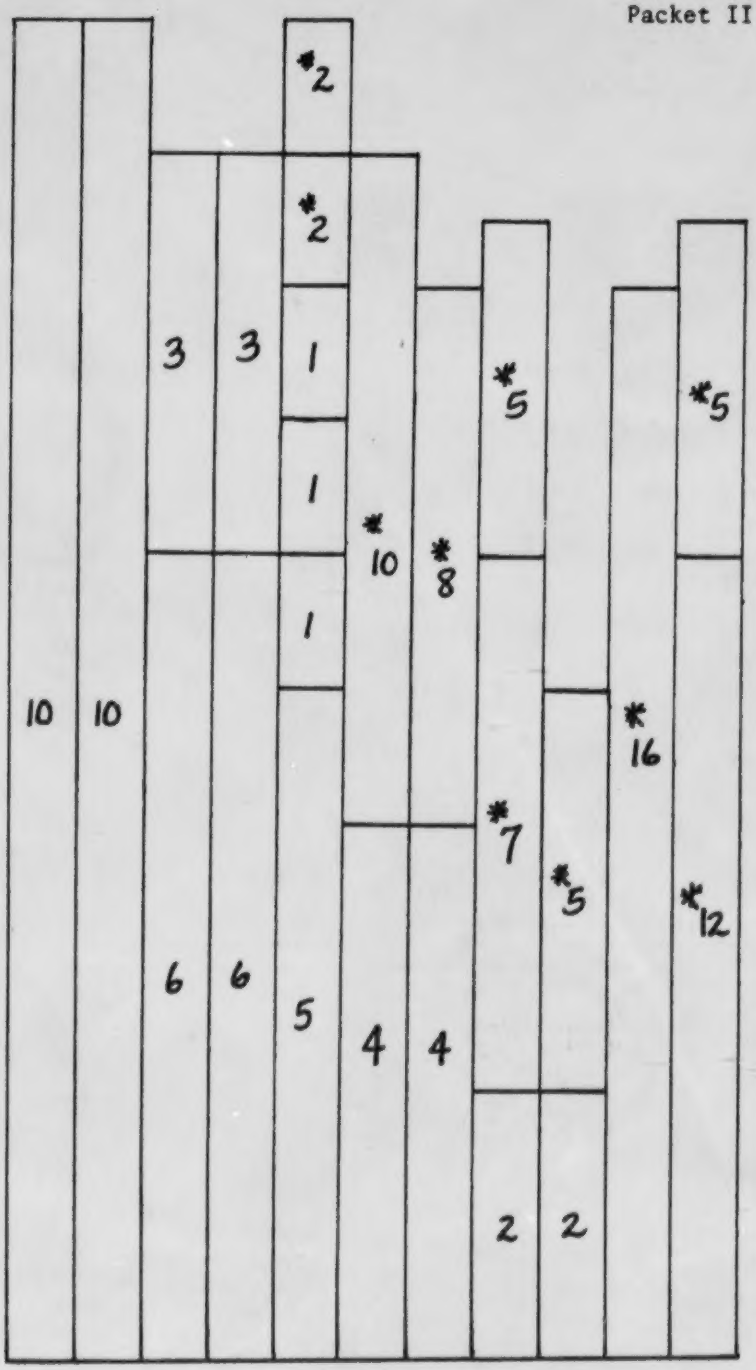
This test involves problem solving.

Review

A review worksheet is provided.

UNIT II TEST

Unit II, Set 5
Packet II


*2
*2
3
3
1
*5
*5
1
*10
*8
1
10
10
*16
*7
*5
*12
6
6
5
4
4
2
2

Directions: Make a ditto master of this sheet.

Ditto copies on 5" by 8" index cards.

Cut the strips and place them into envelopes.

NOTE: Asterisked strips are to be used for Exercise 3 only. nonasterisked strips are to be used for Exercises 1 and 2.

Unit II
Quiz 1

Name ____________________________

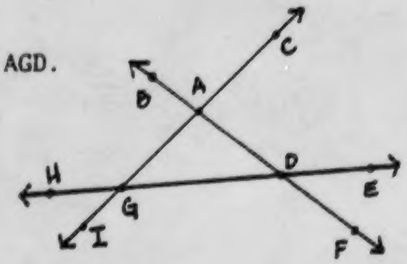
1. Name the longest and shortest sides for $\triangle$ KTV if $m\angle K = 56$, $m\angle T = 23$, and $m\angle V = 101$.

2. If $\angle$ AJC and $\angle$ CJX are adjacent angles, $m\angle AJC = 62$, and $m\angle CJX = 44$, then complete the following by inserting the appropriate symbol--<, >, ≅.

   a. $\angle$ AJC _______ $\angle$ CJX

   b. $\angle$ AJX _______ $\angle$ AJC

3. Name the exterior angles for $\triangle$ AGD.



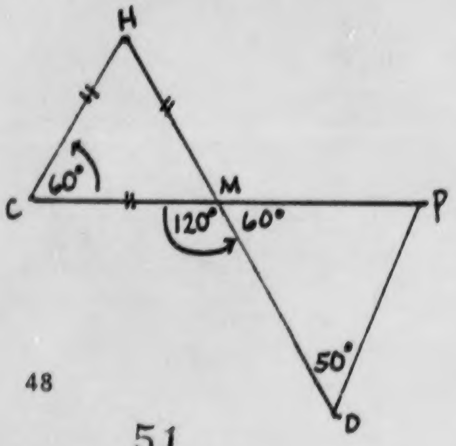
4. Name the largest and smallest angles for $\triangle$ MPT if MP = 10 cm, PT = 4 cm, and MT = 9 cm.

5. The measures of some of the angles are shown in the figure below. Complete the statements with the appropriate symbol--<, >, ≅.

   a. $\angle$ CMD _______ $\angle$ P

   b. $\overline{PD}$ _______ $\overline{PM}$

   c. $\angle$ C _______ $\angle$ CMH



Unit II
Quiz 2

Name ______________________________

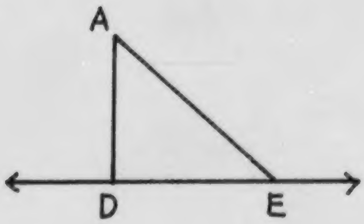
1. Decide whether there is a triangle with the given real numbers as side lengths.

_______ a. 3 5 6

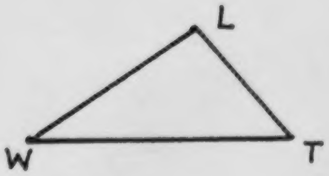
_______ b. 4 9 4

_______ c. 12 15 30

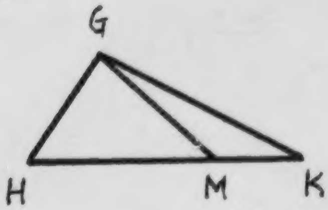
2. $\overline{AD}$ is perpendicular to $\overleftrightarrow{DE}$ as shown. How does $\overline{AE}$ compare with $\overline{DE}$? Explain.



3. If TW > LW > LT, what is the smallest angle of Δ LTW? the largest angle of Δ LTW? Explain.



4. Suppose GM > GH. How is $\angle$GMK related to $\angle$GMH? Explain.



Unit II
Take-home Test

Name ______________________________

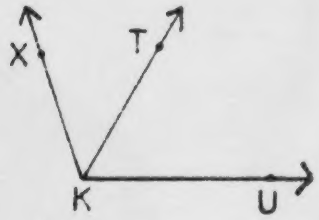
You know already that if in a given triangle, one angle is smaller than another angle, the side opposite the smaller angle is shorter than the side opposite the larger angle. Investigate the following for two triangles:

For the matching $JKL \longleftrightarrow QSV$ between the vertices of two triangles, suppose that $\overline{JK} \cong \overline{QS}$ and $\overline{KL} \cong \overline{SV}$. If $\angle K \not\cong \angle S$, what can you say about $\overline{JL}$ and $\overline{QV}$? Experiment with several pairs of triangles which satisfy the given conditions and state your conclusion.

Unit II
Review Worksheet

Name ______________________

1. Refer to the diagram to answer the questions below:



______ a. m∠ XKT + __?__ = m∠ XKU

______ b. If m ∠ XKT = 50 and m ∠ TKU = 60, then m ∠ XKU = __?__.

2. Tell whether there is a triangle with the given three numbers as side lengths.

______ a. 5 6 8

______ b. 7 4 13

______ c. 10 8 2

3. If your answer was "yes" to any item in Exercise 2 above, write the inequalities which hold among the three given numbers (i.e., write sentences using " > "). If your answer was "no," state your reason using ≯ .

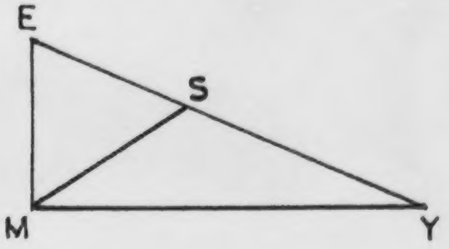
4. In the figure shown, EM = 5, ES = 5, MS = 5, SY = 8, and MY = 12. How do the following compare?

______ a. ∠ E and ∠ EMS

______ b. ∠ MSY and ∠ MYS

______ c. ∠ MSY and ∠ E

______ d. ∠ E and ∠ EMY



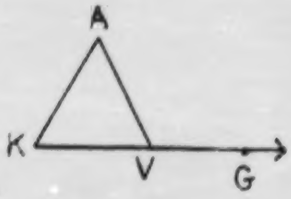
(Go to the next page.)

______ 5. Name the longest and shortest sides for Δ AKV if:

______ $m\angle A = 40$, $m\angle K = 38$, and $m\angle V = 102$.

______ 6. Name the largest and smallest angles for Δ MTW if:

______ MT = 6 cm , TW = 8 cm , and MW = 9 cm



7. $\overline{AK} \cong \overline{AV}$ and $\angle$ AVG is an exterior angle of Δ AKV.

   a. How does $\angle$ AKV compare to $\angle$ AVK? Explain why.

   b. How does $\angle$ AVG compare to $\angle$ AVK? Explain why.

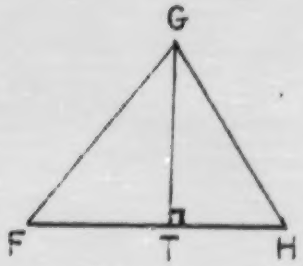
8. If T is a point between points F and M, FM = 19 cm , and FT = 10 cm , then complete the following with the appropriate symbol--<, >, ≅. ≅.

   a. TM _____ FT   b. FM _____ TF

9. Given Δ FGH with FG > GH and $\overline{GT}$ perpendicular to $\overline{FH}$.

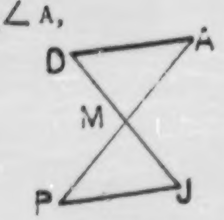
   a. How do $\angle$ H and $\angle$ F compare? Explain why.

   b. How do $\angle$ FTG and $\angle$ H compare? Explain why.



   c. How do $\overline{FG}$ and $\overline{GT}$ compare? Explain why.

10. Suppose M is the intersection of $\overline{AP}$ and $\overline{DJ}$, $m\angle D > m\angle A$, and $m\angle J > m\angle P$ as shown on the right. How does $\overline{DJ}$ compare with $\overline{AP}$?



Unit II Test Name ______________________

1. If G is a point between points K and X, GK = 2 cm, and KX = 15 cm, then complete the following with the appropriate symbol--<, >, ≅.

a. GK ____ GX b. KX ____ GX c. GK ____ KX

2. Name the longest and shortest sides for Δ GHI if:

$m\angle G = 85$, $m\angle H = 35$, and $m\angle I = 60$

____

____

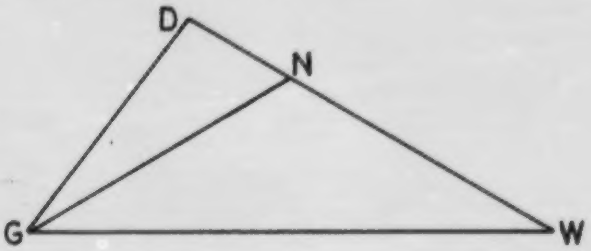
3. In the figure shown, DG = 10, DN = 4, GN = 12, NW = 12, GW = 20. How do the following compare?

____________ a. $\angle D$ and $\angle DGN$

____________ b. $\angle NGW$ and $\angle NWG$

____________ c. $\angle DGW$ and $\angle W$

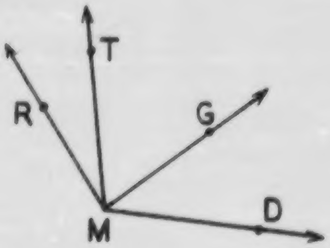
____________ d. $\angle WNG$ and $\angle D$



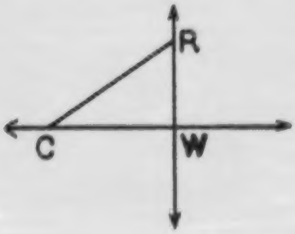
4. Refer to the diagram to answer the questions below:

____________ a. $m\angle RMT + m\angle TMG = m$ __?__

____________ b. If $m\angle DMG = 43$ and $m\angle GMT = 59$, then $m\angle DMT =$ __?__



5. $\overleftrightarrow{RW}$ is perpendicular to $\overleftrightarrow{CW}$ as shown, $m\angle C = 40$, and $m\angle R = 50$. List the sides of ΔCRW from longest to shortest. Then explain the reason for your answer.



(Go on to the next page.)

6. Tell whether there is a triangle with the given three numbers as side lengths.

________ a. 7 4 15

________ b. 12 12 8

________ c. 9 5 4

7. Tell whether each statement is true or false.

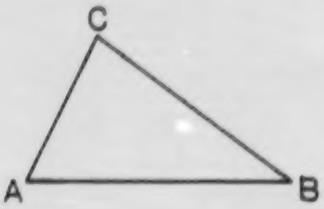
________ a. In $\triangle ABC$, if $m\angle A < m\angle B$ and $m\angle C < m\angle B$, then its greatest side is $\overline{AB}$.

________ b. If an exterior angle of $\triangle ADN$ is congruent to an adjacent interior angle of $\triangle ADN$, then the measure of the exterior angle is 90.

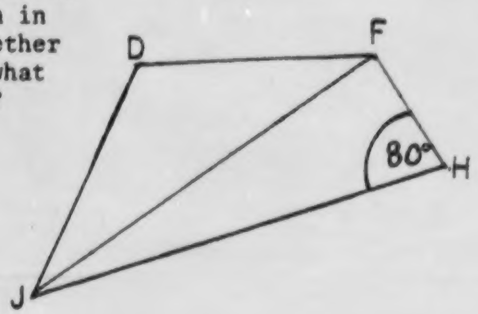
________ c. The shortest segment from a point not on a line to the line is the perpendicular segment.

________ d. If L, P, and T are distinct and collinear points such that $LP = 8$ cm and $LT = 4$ cm, then T can be between L and P.

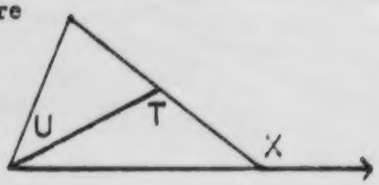
8. Suppose $\triangle ABC$ with $BC > AC$ and $m\angle C > m\angle A$ as shown. How does $\angle C$ compare to $\angle B$? Explain why.



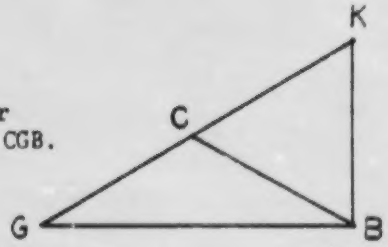
9. Suppose $\overline{JF}$ is the longest side of Δ DFJ and m$\angle$ H = 80° as shown in the diagram. Can you tell whether $\overline{JF}$ or $\overline{JH}$ is longer? If not, what other information do you need?



10. List the angles in order from largest to smallest for the figure shown. Explain the reason for your answer.



11. Suppose Δ BCK is an equiangular triangle. Prove that $\angle$K > $\angle$ CGB.



ANSWER KEY

NOTE: There may be more than one answer for some of the questions.

Quiz 1 (10 *points*)

(2) 1. longest side - $\overline{KT}$
shortest side - $\overline{KV}$

(2) 2. a. $>$ b. $>$

(1) 3. $\angle$ BAG, $\angle$ CAD, $\angle$ ADE, $\angle$ FDG, $\angle$ DGI, $\angle$ AGH

(2) 4. largest angle - $\angle$ T
shortest angle - $\angle$ M

(3) 5. a. $>$ b. $>$ c. $\cong$

Quiz 2 (10 *points*)

(3) 1. a. Yes b. No c. No

(2) 2. $\overline{AE} > \overline{DE}$ since $\angle$ D is the largest angle (it is a right angle) in $\triangle$ ADE.

(3) 3. $\angle$ W is the smallest angle since it is opposite the shortest side of the triangle.
$\angle$ WLT is the largest angle since it is opposite the longest side of the triangle.

(2) 4. $\angle$ GMK $>$ $\angle$ GMH. If GM $>$ GH, then $\angle$ H $>$ $\angle$ GMH by Theorem 7.
$\angle$ GMK is an exterior angle of $\triangle$ GHM. By Theorem 6, $\angle$ GMK $>$ $\angle$ H.
Then by the transitivity property, $\angle$ GMK $>$ $\angle$ GMH.

Take-home Test

$\overline{JL} \not\cong \overline{QV}$.

Review Worksheet

1. a. m $\angle$ TKU b. m $\angle$ XKU = 110

2. a. Yes b. No c. No

3. a. $5 + 6 > 8$, $5 + 8 > 6$, $6 + 8 > 5$
b. $13 \not< 7 + 4$
c. $10 \not< 8 + 2$

4. a. $\cong$ b. $>$ c. $>$ d. $<$

5. longest side: $\overline{AK}$, shortest side: $\overline{AV}$

6. largest angle: $\angle$ T, smallest angle: $\angle$ W

7. a. $\angle$ AKV $\cong$ $\angle$ AVK since $\overline{AK} \cong \overline{AV}$
b. $\angle$ AVG $>$ $\angle$ AVK. $\angle$ AVG $>$ $\angle$ AKV since $\angle$ AVG is an exterior angle of $\triangle$ AKV. Since $\overline{AK} \cong \overline{AV}$, $\angle$ AKV $\cong$ $\angle$ AVK. So $\angle$ AVG $>$ $\angle$ AVK.

(Go on to the next page.)

8. a. $<$ b. $>$

9. a. $\angle H > \angle F$ since $FG > GH$.
   b. $\angle FTG > \angle H$ since $\angle FTG$ is an exterior of ΔGHT.
   c. $\overline{FG} > \overline{GT}$ since $\angle FTG > \angle F$. (From a. and b. above we know that $\angle H > \angle F$ and $\angle FTG > \angle H$. From the transitivity property, it follows that $\angle FTG > \angle F$.)

10. $\overline{DJ} < \overline{AP}$

Unit II Test (*50 points*)

(3) 1. a. $<$ b. $>$ c. $<$

(4) 2. longest side - $\overline{HI}$, shortest side - $\overline{GI}$

(8) 3. a. $\angle D > \angle DGN$ b. $\angle NGW \cong \angle NWG$
c. $\angle DGW > \angle W$ d. $\angle WNG > \angle D$

(4) 4. a. $\angle RMG$ b. 102

(4) 5. From longest to shortest - $\overline{CR}$, $\overline{CW}$, $\overline{RW}$. Since $\overleftrightarrow{RW} \perp \overleftrightarrow{CW}$, $m\angle CWR = 90$. Then $\angle CWR > \angle R > \angle C$. According to Theorem 8, the side opposite the larger angle in a triangle is longer. It follows that $\overline{CR} > \overline{CW} > \overline{RW}$.

(6) 6. a. No b. Yes c. No

(8) 7. a. False b. True c. True d. True

(3) 8. $m\angle C > m\angle B$. Since $BC > AC$, $m\angle A > m\angle B$. Since $m\angle C > m\angle A$ and $m\angle A > m\angle B$, it follows that $m\angle C > m\angle B$ by the transitivity property.

(3) 9. No, we cannot tell whether $\overline{JF}$ or $\overline{JH}$ is longer. We would need to know $m\angle FJH$ or $m\angle JFH$.

(3) 10. From largest to smallest - $\angle X$, $\angle T$, $\angle U$. Since $\angle X$ is an exterior angle, it is larger than $\angle T$, a nonadjacent interior angle. $\angle T$ is an exterior angle and by the same reason it follows that it is larger than $\angle U$.

(4) 11. Suppose ΔBCK is an equiangular triangle. Then by the definition of an equiangular triangle, $\angle KCB \cong \angle KBC \cong \angle K$. $\angle KCB$ is an exterior angle of ΔBCG by the definition of an exterior angle. Then $\angle KCB > \angle CGB$ by Theorem 6. Since $\angle KCB \cong \angle K$, $\angle K > \angle CGB$.

Unit III: Parallelism and Other Relations in Planes and Space

Objectives

(1) To discover relations among points, lines, and planes.

(2) To identify alternate interior angles, alternate exterior angles, corresponding angles, interior angles on the same side of the transversal, and exterior angles on the same side of the transversal.

(3) To discover and use the properties of parallel lines cut by a transversal.

(4) To prove that lines are parallel.

(5) To construct parallel lines.

(6) To prove and apply the angle sum theorem for a triangle.

(7) To discover and use the properties of a parallelogram.

(8) To discover and use the relationship that if three parallel lines intersect two transversal, they divide the transversals proportionally.

Materials Needed

Set 1: a sheet of paper

Set 3: compass, straightedge, protractor

Set 4: protractor, ruler

Suggestions for Teaching

Set 1, pages 84 - 85

Vocabulary: plane, intersect, parallel, space

Symbols: $\ell_1 || \ell_2$

Materials Needed: sheet of paper

Comments: This set develops students' awareness of geometric relations in two and three dimensions. The relationships between two or more lines in a plane, two or more lines in space, a line and a plane in space, and so on, are explored.

The classroom provides many examples of lines and planes in space. Discuss these with the students. Be sure to remind students that lines and planes extend indefinitely. Physical things around us are only used to suggest them.

"SKEW" LINES

The name "skew" lines may be introduced in discussing Exercise 2, but it is not necessary. The essential idea is that in space there are lines which are not parallel and not intersecting.

Set 2, pages 86 - 88

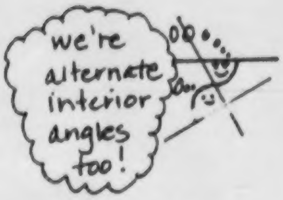
Comments: Encourage students to use physical objects, such as pencils to represent lines and plastic or cardboard sheets to represent planes, to see the interrelationship between lines and planes. Having students do this in groups would be more valuable than teacher demonstrations.

Set 3, pages 89 - 92

Vocabulary: transversal, interior angle, exterior angle, alternate interior angle, corresponding angle, alternate exterior angle

Materials Needed: compass, straightedge, protractor

Comments: Although parallel lines will be the main concern, students should recognize that nonparallel lines can also be cut by a transversal to form the alternate interior angles, the corresponding angles, and so on.


we're alternate interior angles too!

PROBLEM SOLVING: *Finding Patterns*

An important problem-solving skill is to be able to find patterns in a set of data. In Exercise 6 on page 91, students must measure the various angles formed by parallel lines cut by a transversal to see if they can find any patterns. Have students discuss their results. Describing patterns is also a skill that needs to be developed and encouraged.

Students may need some help with the constructions in Exercise 8 on page 91. You may want to provide them with a hint: Construct a copy of an angle.

PROBLEM SOLVING

For Exercise 9, encourage students to use the four steps to problem solving. They should ask themselves questions such as:

---What is the problem?

---What information is being given?

---How can the fact that line L is parallel to $\overleftrightarrow{AB}$ be used? That is, what do they know about parallel lines?

QUIZ 1

Anytime after Set 3

Set 4, pages 93 - 94

Vocabulary: parallelogram, opposite sides, opposite angles, consecutive angles, diagonal

Materials Needed: protractor, ruler

Comments: Have students explain how they obtained their conclusions for Exercise 1 on page 93.

In the discussion of Exercise 1, be sure to point out:

(1) no measurement is completely accurate, and
(2) it would be impossible to test every parallelogram since there is an unlimited number of them.

Set 5, pages 95 - 97

Comments: Have students discuss their answers to Exercises 1 through 4 as a class before answering the rest of the exercises.

For additional practice, assign Supplementary Worksheet 1.

QUIZ 2

Anytime after Set 5

Set 6, pages 98 - 107

Comments: As in the previous units, have students discuss the new definitions, postulates, and theorems.

PROOFS

The proofs should be read and discussed with students. Remember that the proofs are primarily intended to expose students to and acquaint them with correct mathematical language and presentation rather than "mastery" and proficiency in constructing proofs.

An indirect proof is introduced to prove Theorem 11 on pages 99 and 100. Review this carefully with the students.

Review

A review worksheet is provided for this unit.

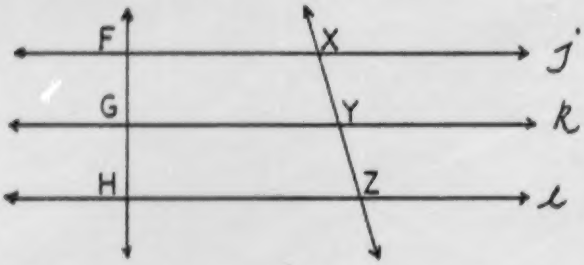
UNIT III TEST

Unit III
Supplementary Worksheet I

Name ______________________

Date ______________________

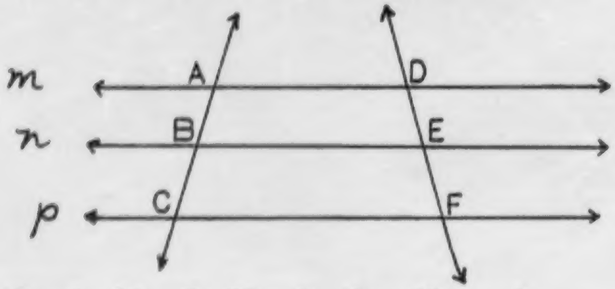
Suppose $j \parallel k \parallel \ell$ in the figure below. Find the designated lengths without measuring.



______ 1. If FG = 8, GH = 10, YZ = 15, then XY = __?__

______ 2. If $\overline{FG} \cong \overline{GH}$ and XY = 10, then YZ = __?__

Suppose $m \parallel n$ and $n \parallel p$ in the figure below. Find the designated lengths without measuring.



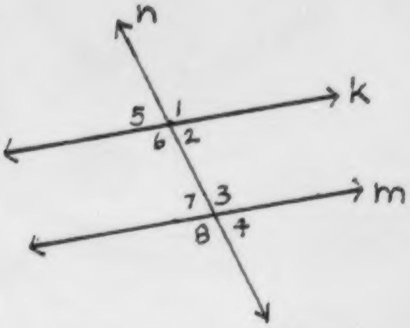
______ 3. If AB = 3, DE = 2, EF = 5, then BC = __?__

______ 4. If BC = $\frac{9}{2}$, DE = 6, EF = 3, then AB = __?__

______ 5. If AB = 1, BC = 4, DE = $\frac{7}{2}$, then EF = __?__

Unit III
Quiz 1

Name ____________________



Suppose k ‖ m with transversal n.
Answer questions 1 - 6 below.

____ 1. Name a pair of alternate interior angles.

____ 2. Name a pair of corresponding angles.

____ 3. If m∠6 = 120 , then m∠8 = ? .

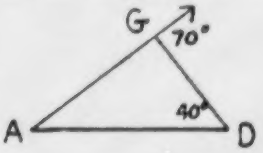
____ 4. If m∠2 = 80 , then m∠3 = ? .

____ 5. Name a pair of interior angles on the same side of the transversal.

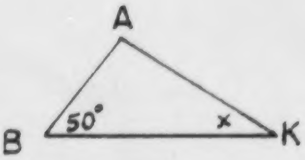
____ 6. Name an angle congruent to ∠5. Explain why the angles are congruent.

7. Find the measure of ∠A in each diagram below:

____ a.



____________ b.

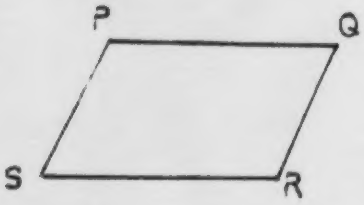


8. In a plane if two lines are perpendicular to the same line, do you think they must be parallel? Explain your answer.

Unit III
Quiz 2

Name ______________________________

1. Suppose PQRS is a parallelogram with PQ = 6, PS = 4, and $m\angle S = 65$. Find each length and angle measure below:

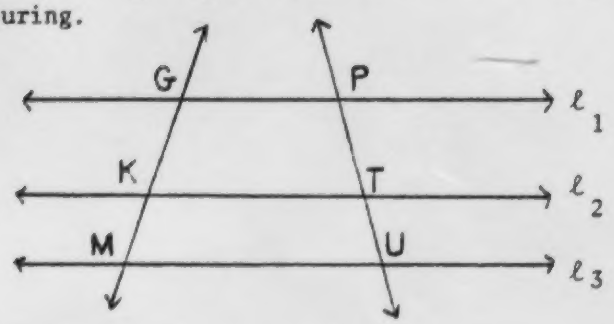


a. RS = ______ c. $m\angle R$ = ______

b. QR = ______ d. $m\angle Q$ = ______

2. How long are the sides of a parallelogram if one of its sides is 3 centimeters longer than the other and the perimeter is 54 centimeters?

3. Suppose $\ell_1 \parallel \ell_2$ and $\ell_2 \parallel \ell_3$ as below. Find the designated lengths without measuring.



a. If GK = 16, KM = 4, TU = 3, then PT = ______.

b. If KM = 7, PT = 18, TU = 6, then GK = ______.

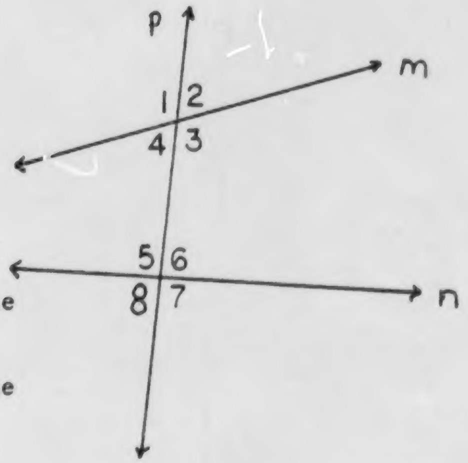
c. If PT = 6, TU = 1, and KM = $\frac{3}{2}$, then GK = ______.

d. If GK = 6, TU = 2.5, PT = 10, then KM = ______.

Unit III
Review Worksheet

Name ______________________________

1. Refer to the figure shown on the right to answer the following:



________ a. Identify the transversal.

________ b. Name a pair of alternate interior angles.

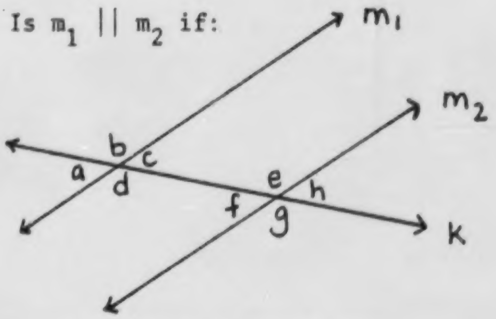
________ c. Name a pair of corresponding angles.

________ d. Name a pair of interior angles on the same side of the transversal.

________ e. Name a pair of exterior angles on the same side of the transversal.

________ f. Name a pair of alternate exterior angles.

2. Refer to the figure shown. Is $m_1 \parallel m_2$ if: (Answer yes or no.)



________ a. a = 100 and f = 100

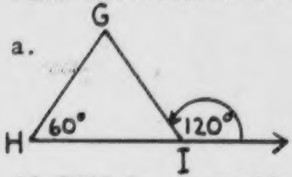
________ b. c = 80 and g = 80

________ c. b = 40 and h = 140

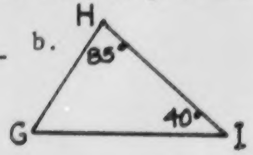
________ d. d + f = 180

3. Find the measure of ∠ G in each diagram shown below:

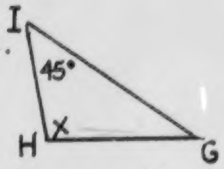
____ a.



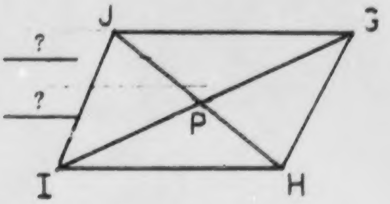
____ b.



____ c.



4. If GHIJ is a parallelogram, m ∠ G = 70, GJ = 14, GH = 10, GI = 18, JH = 12, then:



________ a. m ∠ H = __?__

________ b. m ∠ I = __?__

________ c. HI = __?__

________ d. JP = __?__

________ e. IJ = __?__

(Go on to the next page.)

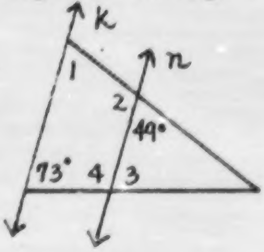
5. Refer to each diagram shown in which $k \parallel n$. Find the measures of angles 1 through 6.

____ a.

____

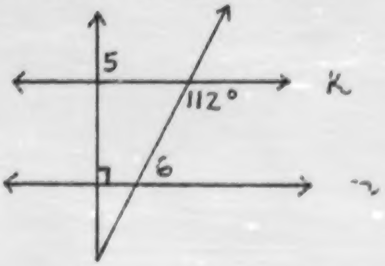
____

____



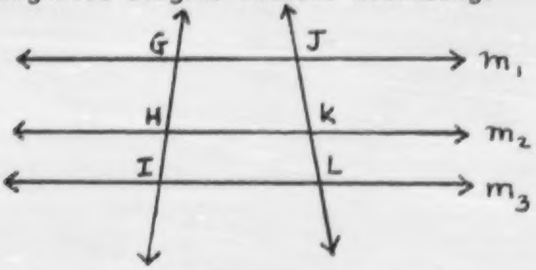
____ b.

____



6. Draw figures showing how two lines in space may be related to each other.

7. Suppose $m_1 \parallel m_2$ and $m_2 \parallel m_3$ in the figure shown below. Find the designated lengths without measuring.



____ a. If GH = 12, HI = 4, KL = 5, then JK = __?__.

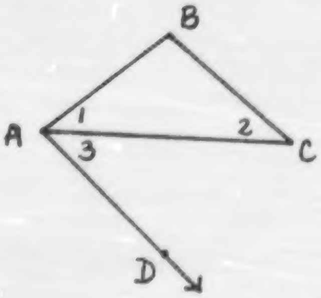
____ b. If JK = 10, KL = 6, GH = 15, then HI = __?__.

____ c. If HI = $\frac{5}{2}$, JK = 8, KL = 4, then GH = __?__.

____ d. If GH = 14, HI = 12, JK = 21, then KL = __?__.

8. Suppose $\overline{AB} \cong \overline{BC}$ and $\angle 1 \cong \angle 3$ as shown.

Prove that $\overline{BC} \parallel \overline{AD}$.



Unit III Test   Name ____________________

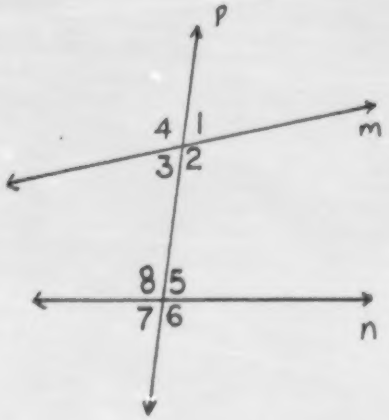
1. Refer to the figure on the right to answer the following:

______ a. Identify the transversal.

______ b. Name a pair of interior angles on the same side of the transversal.

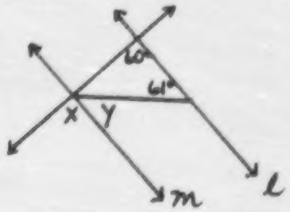
______ c. Name a pair of corresponding angles.

______ d. Name a pair of alternate interior angles.

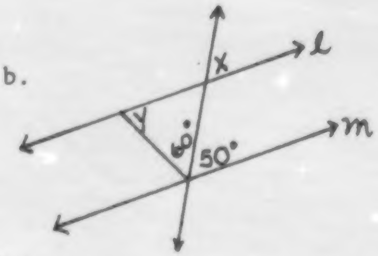

p
m
n
4
1
3
2
8
5
7
6

2. Refer to the diagrams shown in which $\ell \parallel m$. Find the value of angles x and y in each case.

x = ______
y = ______
a.



x = ______
y = ______
b.

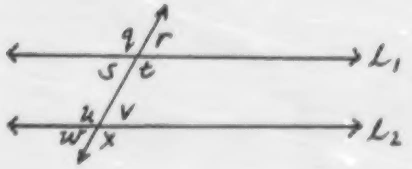


3. Refer to the figure shown. Is $\ell_1 \parallel \ell_2$ if: (Answer yes or no.)

______ a. r = 80° and w = 80°

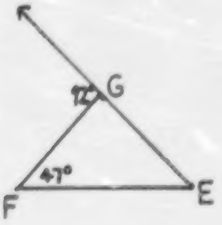
______ b. q = 140° and w = 60°

______ c. s + u = 180°

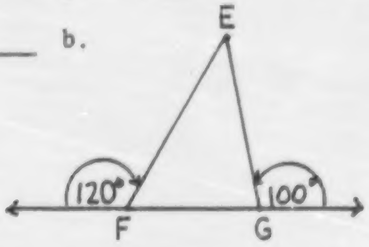

q
r
s
t
u
v
w
x
ℓ_1
ℓ_2

4. Find the measure of ∠ E in each diagram below:

______ a.

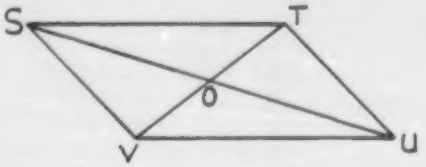


______ b.



(Go on to the next page.)

5. STUV is a parallelogram.



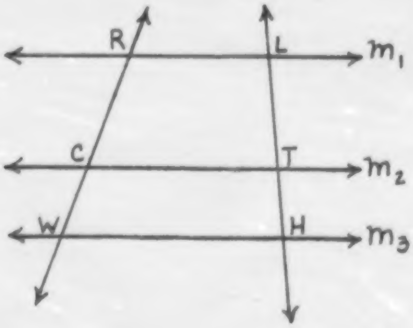
______ a. If m $\angle$ TSV = 50 ,
then m $\angle$ TUV = ?

______ b. If TV = 10, then
OV = ?

______ c. If SV = 8, then
TU = ?

______
______ 6. If one of two consecutive angles of a parallelogram is twice the other, what is the degree measure of each of the angles?

7. Suppose $m_1 \parallel m_2 \parallel m_3$ in the figure shown. Find the designated lengths without measuring.



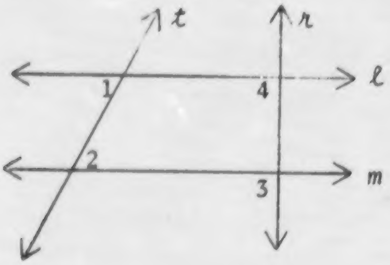
______ a. If CR = 18, HT = 4, CW = 3,
then LT = ? .

______ b. If HT = 5, CR = 20, LH = 30,
then CW = ? .

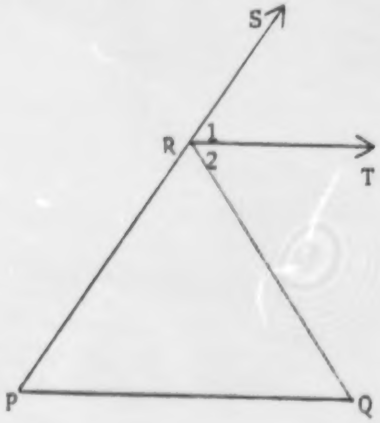
______ c. If CW = 4, CR = 14, HT = 2,
then LT = ? .

8. Draw figures showing how three lines in a plane may be related to each other.

9. a. Lines ℓ and m are cut by transversals t and r. If $\angle 1 \cong \angle 2$ and $r \perp \ell$, what is $m\angle 3$? Why?



10. Suppose that $\overleftrightarrow{RT} \parallel \overline{PQ}$ and $\angle P \cong \angle 2$ in the figure shown below. What kind of triangle is ΔPQR? Why?



ANSWER KEY

NOTE: There may be more than one answer for some of the questions.

Supplementary Worksheet 1

1. XY = 12 2. YZ = 10 3. BC = 7.5

4. AB = 9 5. EF = 14

Quiz 1 (10 points)

(1) 1. $\angle$ 2 and $\angle$ 7 OR $\angle$ 3 and $\angle$ 6

(1) 2. $\angle$ 1 and $\angle$ 3, $\angle$ 2 and $\angle$ 4, $\angle$ 5 and $\angle$ 7, OR $\angle$ 6 and $\angle$ 8

(1) 3. $m\angle 8 = 120$

(1) 4. $m\angle 3 = 100$

(1) 5. $\angle$ 2 and $\angle$ 3 OR $\angle$ 6 and $\angle$ 7

(1) 6. Answers may vary. Here are some possible answers:

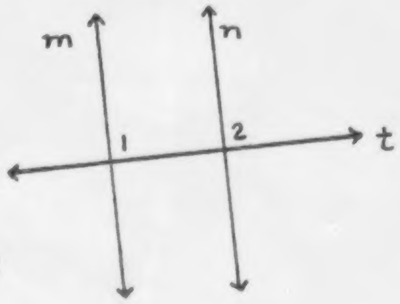
1) $\angle 2 \cong \angle 5$ since $\angle$ 2 and $\angle$ 5 are vertical angles and vertical angles are congruent.

2) $\angle 7 \cong \angle 5$ since $k \parallel m$ and $\angle$ 7 and $\angle$ 5 are corresponding angles.

3) $\angle 4 \cong \angle 5$ since $k \parallel m$ and $\angle$ 4 and $\angle$ 5 are alternate exterior angles.

(2) 7. a. $m\angle A = 30$ b. $m\angle A = 180 - (50 + x) = 130 - x$

(2) 8. Yes, the lines are parallel. For example, suppose lines m and n are perpendicular to line t. By the definition of perpendicular lines, $\angle$ 1 and $\angle$ 2 are right angles. Then $\angle 1 \cong \angle 2$. $\angle$ 1 and $\angle$ 2 are corresponding angles by the definition of corresponding angles. It follows that $m \parallel n$ by theorem 12.



(Go on to the next page.)

Quiz 2 *(10 points)*

(4) 1. a. RS = 6 b. QR = 4 c. $m\angle R = 115$ d. $m\angle Q = 65$

(2) 2. width = 12 cm, length = 15 cm

(4) 3. a. PT = 12 b. GK = 21 c. GK = 9 d. KM = 1.5

Review Worksheet

1. a. line p

   b. $\angle 3$ and $\angle 5$ OR $\angle 4$ and $\angle 6$

   c. $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$, OR $\angle 3$ and $\angle 7$

   d. $\angle 3$ and $\angle 6$ OR $\angle 4$ and $\angle 5$

   e. $\angle 1$ and $\angle 8$ OR $\angle 2$ and $\angle 7$

   f. $\angle 1$ and $\angle 7$ OR $\angle 2$ and $\angle 8$

2. a. Yes b. No c. Yes d. Yes

3. a. $m\angle G = 60$ b. $m\angle G = 55$

   c. $m\angle G = 180 - (45 + x) = 135 - x$

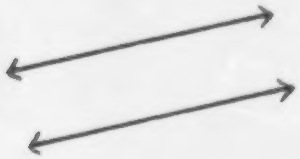
4. a. $m\angle H = 110$ b. $m\angle I = 70$ c. HI = 14

   d. JP = 6 e. IJ = 10

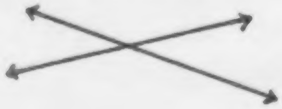
5. a. $m\angle 1 = 49$, $m\angle 2 = 131$, $m\angle 3 = 73$, $m\angle 4 = 107$

   b. $m\angle 5 = 90$, $m\angle 6 = 68$

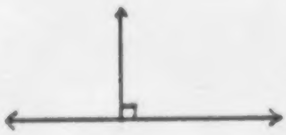
6. 1) lines may be parallel 2) lines may intersect



a)

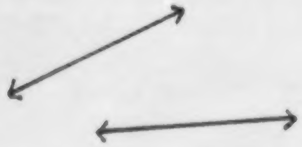


b)



(Go on to the next page.)

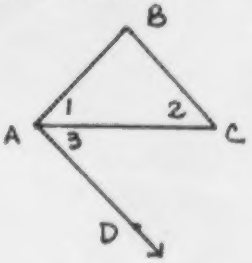
3) lines may be neither parallel nor intersecting



7. a. JK = 15 b. HI = 9 c. GH = 5 d. KL = 18

8. Answers will vary. Here is a possible answer:

Suppose $\overline{AB} \cong \overline{BC}$ and $\angle 1 \cong \angle 3$ as shown. Since $\overline{AB} \cong \overline{BC}$, then $\angle 1 \cong \angle 2$ by Theorem 2. By the transitivity property, it follows that $\angle 2 \cong \angle 3$. $\overline{BC} \parallel \overline{AD}$ by Theorem 11.



Unit III Test (50 *points*)

(1) 1. a. line p

(2) b. ∠ 2 and ∠ 5 OR ∠ 3 and ∠ 8

(2) c. ∠ 1 and ∠ 5, ∠ 2 and ∠ 6, ∠ 3 and ∠ 7, OR ∠ 4 and ∠ 8

(2) d. ∠ 2 and ∠ 8 OR ∠ 3 and ∠ 5

(8) 2. a. x = 60°, y = 61° b. x = 50°, y = 70°

(6) 3. a. Yes b. No c. Yes

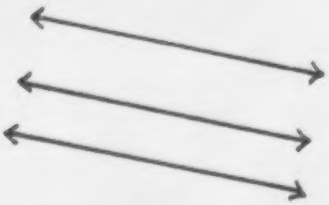
(4) 4. a. m ∠ E = 45 b. m ∠ E = 40

(6) 5. a. m ∠ TUV = 50 b. OV = 5 c. TU = 8

(3) 6. One angle has degree measure 60; the other angle has degree measure 120.

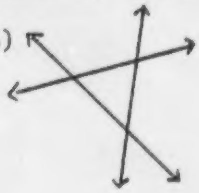
(6) 7. a. LT = 24 b. CW = 4 c. LT = 7

(2) 8. 1) lines may be parallel

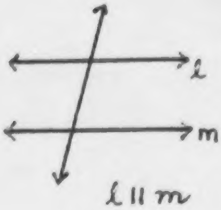


2) lines may intersect

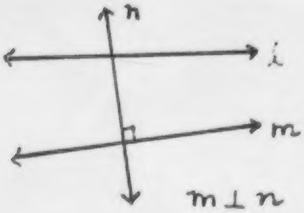
a)



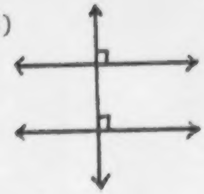
b)



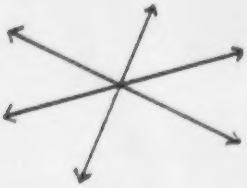
c)



d)



e)



(4) 9. m ∠ 3 = 90. Since ∠ 1 ≅ ∠ 2 and ∠ 1 and ∠ 2 are alternate interior angles, lines ℓ and m must be parallel. Since $n \perp \ell$ then ∠ 4 is a right angle or has a measure of 90. Since $\ell \parallel m$, it follows that ∠ 4 and ∠ 3 are congruent corresponding angles. Therefore, m ∠ 3 = 90.

(4) 10. △ PQR is an isosceles triangle. Since $\overrightarrow{RT} \parallel \overline{PQ}$ and ∠ 2 and ∠ Q are alternate interior angles, ∠ 2 ≅ ∠ Q. Then since ∠ P ≅ ∠ 2, it follows that ∠ P ≅ ∠ Q by the transitivity property. Since ∠ P ≅ ∠ Q, $\overline{PR} \cong \overline{QR}$ by the theorem which states that sides opposite the congruent angles of a triangle are congruent (Theorem 3). From the definition of an isosceles triangle, we can conclude that △ PQR is an isosceles triangle.

Unit IV: Similarity

Objectives

(1) To recognize and apply the definition and the properties of similar triangles.

(2) To find missing parts of similar triangles without measuring.

(3) To investigate and apply the SAS, SSS, and AA Similarity Postulates.

(4) To investigate and use the theorem that given ΔPQR, if S is between P and R, and T is between Q and R, and $\overline{ST} \parallel \overline{PQ}$, then $\Delta PQR \sim \Delta STR$.

(5) To investigate and use the theorem that the altitude from the vertex of the right angle of a triangle forms two right triangles each of which is similar to the given triangle.

(6) To investigate and use the theorem that the square of the altitude from the vertex of the right angle of a triangle is equal to the product of the two segments which are formed when the altitude divides the hypotenuse.

(7) To investigate and use the Pythagorean Theorem and its converse.

(8) To solve real-world problems using similar triangles.

Materials Needed

Set 2: compass and straightedge

Suggestions for Teaching

Set 1, pages 110 - 114

Vocabulary: similar

Symbols: $\sim$

Comments: The definition of similarity and the properties of similar triangles are developed here. Students should see that two conditions are needed for similarity. The teacher should stress how congruent and similar figures are alike and how they are different; it will make the definitions, postulates, and theorems about similarity easier to remember.

"REAL" EXAMPLES

You may want to have students discuss examples of similar objects in our environment. This will emphasize that the examples of similarity are plentiful around us.

CHECK FOR REASONABLENESS

In doing exercises like exercises 11 - 13 on pages 112 - 113, remind students to check the reasonableness of their answers.

Set 2, pages 115 - 117

Comments: Students are asked to explore the possibility of knowing about fewer than six pairs of parts in order to know that two triangles are similar. Exercises 1 and 2 enable them to explore the SAS Similarity. Exercises 3 and 4 and Exercises 5 and 6 enable students to explore the SSS Similarity and AA Similarity, respectively.

WORK WITH THE STUDENTS

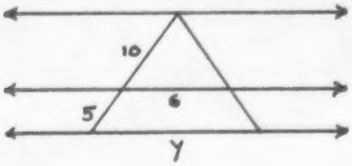
You may want to have the students work in pairs to do Exercises 1 through 7. It is recommended that you work with the students as they do the assignment. Help them to read, reread, and follow directions, instead of demonstrating how to do the constructions.

Come together as a class to discuss the students' results for the exercises. Have students explain reasons for their selections of similar triangles on page 117. (A transparency master of page 117 is provided.)

Set 3, pages 118 - 121

Vocabulary: right triangle, altitude

Comments: Students commonly make the error of using segments which are not sides of triangles but only parts of sides.



Common error:

$\frac{10}{5} = \frac{6}{y}$, so $y = 3$

In Exercise 9 on page 120, students are asked to explore the relationship between the altitude from the vertex of a right triangle and the segments which are formed when the altitude divides the hypotenuse. You may need to provide the students with a hint:

$$\Delta\ DCB \sim \Delta\ DBA, \text{ then } \frac{DC}{DB} = \frac{DB}{?}$$

Be sure to have the students discuss their results for Exercises 8 through 10 before they continue with the rest of the exercises.

QUIZ 1

Anytime after Set 3

Set 4, pages 122 - 123

Symbols: b_1 , a^2

Comments: Exercises 1 - 5 are the steps for an algebraic proof of the Pythagorean Theorem, involving ratios of lengths of sides of similar right triangles. Historically, the proof given of the Pythagorean Theorem involved areas. After students have studied area in Unit VII, you may want to discuss this proof.

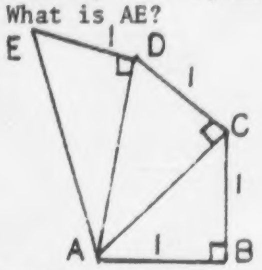
Be sure to discuss Exercise 6 on page 123. Students should realize that the statement is the converse of the Pythagorean Theorem. They should also realize that it must be proven as a separate theorem. Discuss the proof as a class.

REVIEW SQUARE ROOTS

Students may need a review of square roots to do some of the problems in Exercises 7 and 8.

Here is an activity for the students to use the Pythagorean Theorem to construct square roots of the whole numbers.

1) Draw a right triangle ABC, with the right angle at B and AB = BC = 1. By the Pythagorean Theorem, what is the length of the hypotenuse $\overline{AC}$?
2) Now draw Δ ACD with a right angle at C, D and B on opposite sides of $\overline{AC}$, and CD = 1. What is AD?
3) Next, draw Δ ADE with a right angle at D, DE = 1, and E and C on opposite sides of $\overline{AD}$. What is AE?
4) Continue in the same way, until you have constructed a segment whose length is $\sqrt{7}$.



Set 5, pages 124 - 125

Comments: Students are asked to do some similarity applications. Be sure to have students discuss how they obtained their answers.

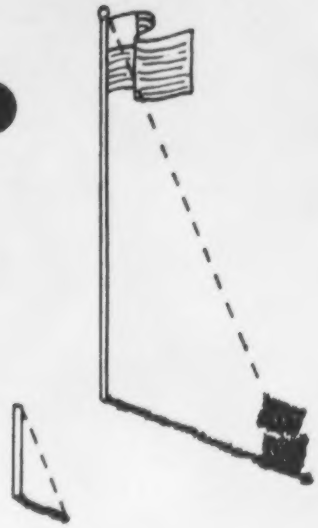
EXPERIENCES WITH SIMILARITY

If time permits, students should do one or two activities which provide them with actual physical experience involving similarity. The experience gained from doing these activities will put more meaning and understanding into the "paper and pencil" exercises in the textbook.

Here are a few suggested activities:

Activity 1

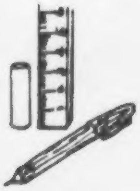
Materials: two meter sticks



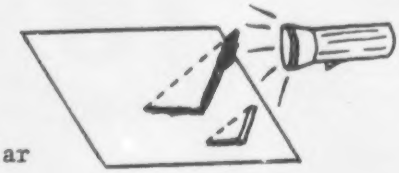
1. To find the height of the flagpole in your schoolyard, follow the directions below:
   a. Place one meter stick in line with the flagpole. Using the other meter stick, measure its shadow to the nearest centimeter.
   b. Measure the shadow of the flagpole to the nearest centimeter.
   c. Do you see that you have two similar right triangles? Make a sketch of this in your notebook.
   d. Find the height of the flagpole.
2. Find the height of a tree in your schoolyard. Follow directions similar to that for Problem 1.
3. Find the height of an object of your choice.

Activity 2 (Work with a partner.)

Materials: flashlight, paper, two objects of different lengths (example: a pencil and a piece of chalk)

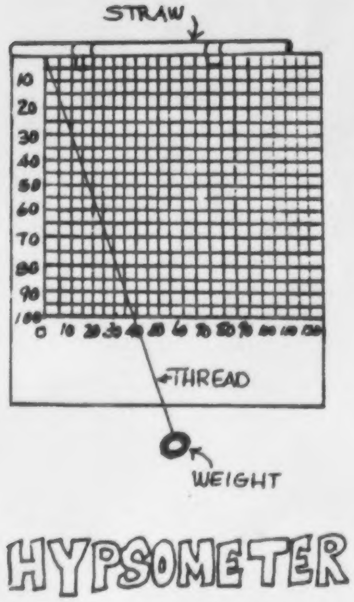


1. Measure the height of the shorter object to the nearest centimeter.
2. Place the two objects in an upright position on a piece of paper, so that both objects form right angles with the paper. (You may have to hold them!)
3. Have your partner shine the flashlight so that each object casts its shadow completely onto the paper. Quickly mark off the beginning and end of each shadow. Do you see that you have two similar right triangles?



4. Measure the lengths of the shadows of both objects to the nearest centimeter.
5. In your notebook, make a sketch of the two similar right triangles formed. On your sketch, indicate the length of the sides you have measured.
6. Without measuring, find the height of the taller object.
7. Now measure the height of the taller object. How close was your answer to Step 6 above?

Activity 3 (Work with a partner.)


STRAW
10
20
30
40
50
60
70
80
90
100
0
THREAD
WEIGHT
HYPSOMETER

1. Follow the directions below to make a hypsometer.

HOW TO MAKE A HYPSOMETER

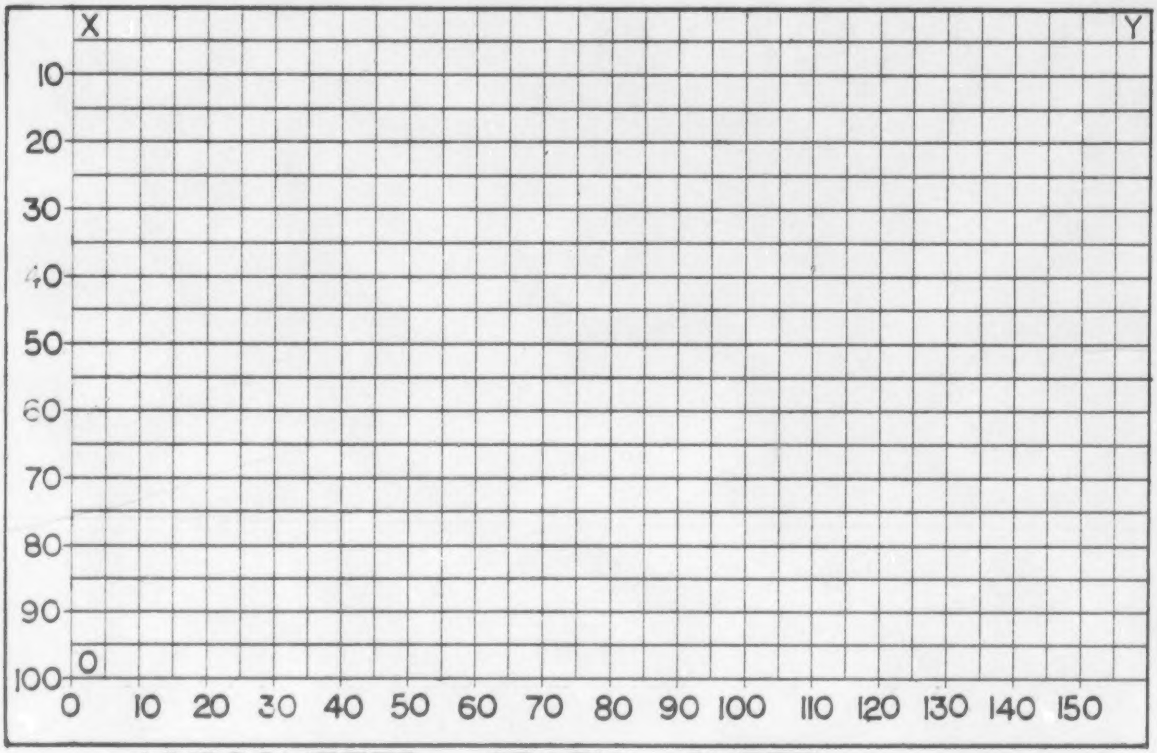
Materials: piece of cardboard, hypsometer graph sheet, soda straw, thread, washers or small weights, straight pins, and tape

1) Tape the hypsometer graph sheet onto the cardboard so that the edge $\overline{XY}$ is in line with the top edge of the cardboard.

2) Tape a soda straw along the top edge of the cardboard.

3) Tie a piece of thread to a pin at point X in the upper left-hand corner.

4) Tie a weight to the other end of the thread so that it will hang vertically.


X
Y
10
20
30
40
50
60
70
80
90
100
O
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

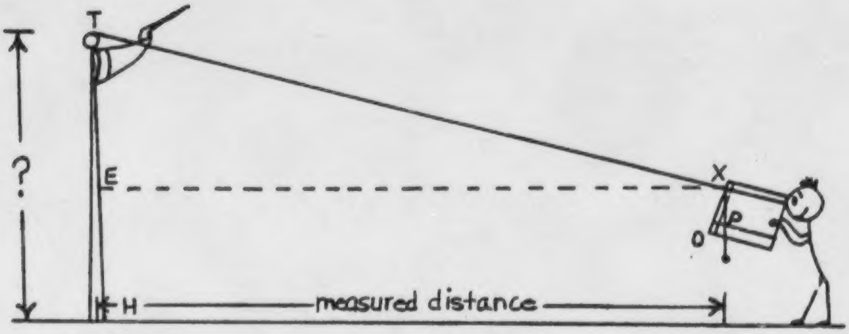
HYPSOMETER GRAPH SHEET

2. Follow the directions below to help you use your hypsometer to find the following:

   a. the height of the classroom

   b. the height of the door

HOW TO USE A HYPSOMETER



1) Stand at a convenient distance from an object whose height you wish to measure. Measure this distance.

2) Sight the top of the object through the straw.

3) Mark the point where the thread intersects the horizontal scale of the hypsometer.

4) Triangles EXT and OXP are similar. Draw a picture of the two similar triangles, with their known measures, in your notebook.

5) Find the height of $\overline{ET}$.

6) Find the height of your eye level (HE) above the ground level.

7) The total height of the object includes the height HE plus the height of your eye above ground level:

   Total height of the object = ET + HE

3. Go outside and use the hypsometer to find the height of tall objects.

(Activities are taken from the similarity unit of the Intermediate School Mathematics Program.)

QUIZ 2

Anytime after Set 5

Set 6, pages 126 - 135

Vocabulary: hypotenuse

Comments: Formal definitions, postulates, and theorems are presented. Have students review these carefully.

Assign the Take-home Test.

Review

A review worksheet is provided.

UNIT IV TEST

Transparency
Unit IV Set 2

7. On the basis of the preceding exercises, state the conditions under which two triangles are similar.

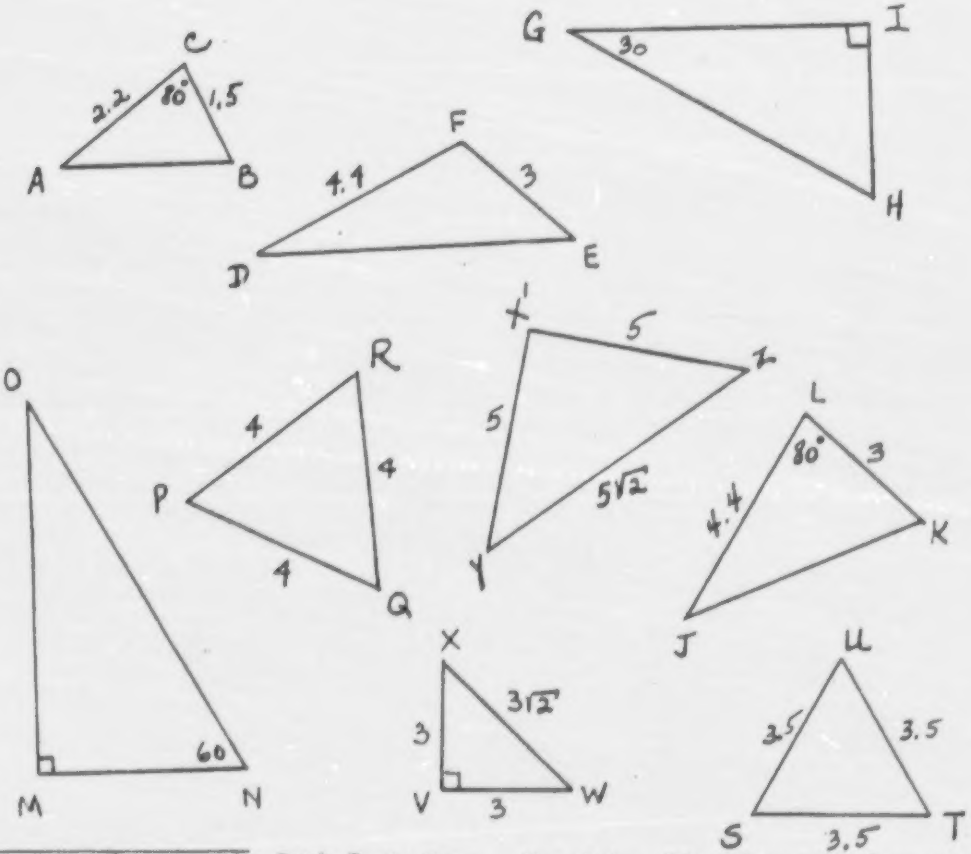
   Are there conditions similar to the SAS congruence conditions? SSS?

   Why would there not be a set of conditions analogous to ASA congruence?

8. Are there certain types of triangles for which there is more than one matching between the vertices which shows similarity? Explain.

9. Name some figures which are always similar.

On the basis of these exercises and the designated measures, select pairs of similar triangles shown below.

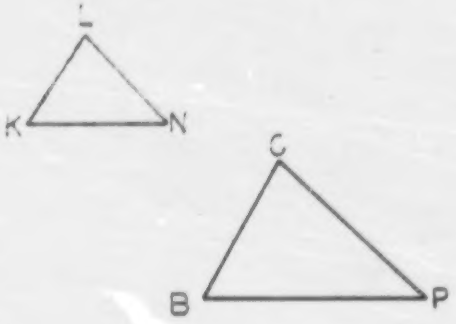

C
2.2
80°
1.5
A
B
G
30
I
H
F
4.4
3
D
E
O
M
N
60
R
4
4
P
4
Q
X'
5
Z
5
5√2
Y
L
80°
3
4.4
K
J
X
3√2
3
V
3
W
U
3.5
3.5
S
3.5
T

From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 117.

Unit IV
Quiz 1

Name ______________________

1. Given Δ KLN ~ Δ BCP as shown.

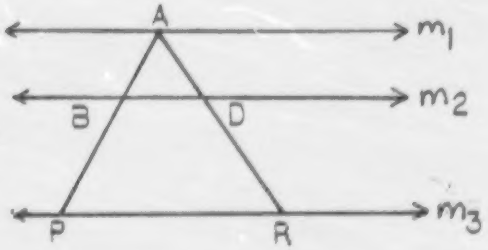


______ a. If m ∠ K = 60 and m ∠ N = 35, find m ∠ B and m ∠ C.

______

______ b. If KL = 4, LN = 5, and BC = 10, find CP.

2. On the right are parallel lines m_1, m_2, m_3, cut by transversals to form triangles ABD and APR. If m ∠ ADB = 50, AB = 4, AP = 12, AD = 5, and BD = 7, find:

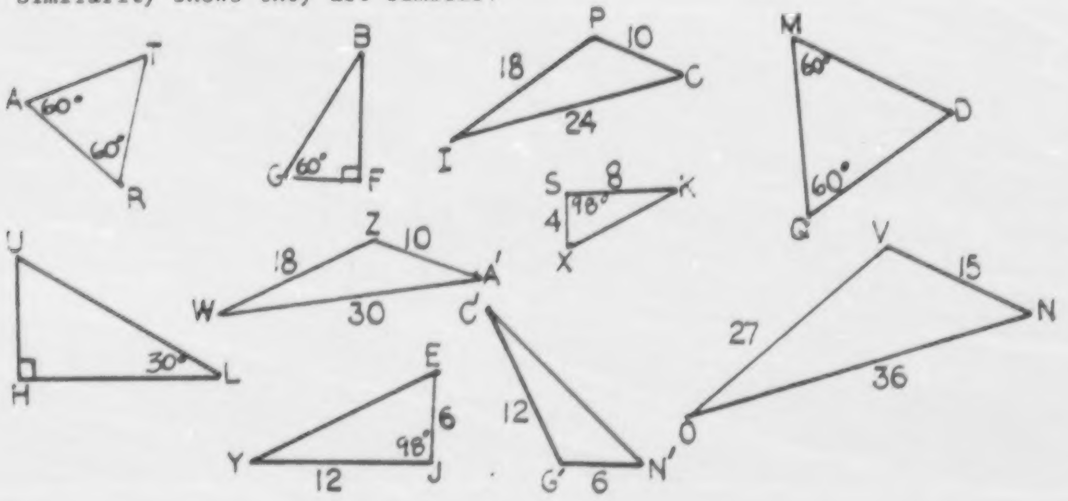

A
m_1
B
D
m_2
P
R
m_3

______ a. m ∠ARP = ___?___

______ b. AR = ___?___

______ c. PR = ___?___

3. Name the pairs of similar triangles. State which of SAS, SSS, or AAA Similarity shows they are similar.


T
A
60°
60°
R
B
G
60°
F
P
10
18
C
24
I
M
60°
D
60°
Q
S
8
K
98°
4
X
U
H
30°
L
Z
18
10
A′
W
30
C′
V
15
N
27
36
O
E
6
98°
Y
12
J
12
G′
6
N′

Unit IV
Quiz 2

Name ______________________________

1. Complete the following, using the Pythagorean Theorem.

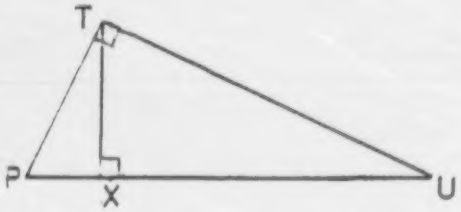
   a. Suppose ∠ C of Δ CJK is a right angle, CK = 6 and CJ = 8. Find JK.

   b. Suppose ∠ W of Δ LWX is a right angle, WX = 12 and LX = 20. Find LW.

2. Is Δ DHO a right triangle if DH = 9, HO = 15, and DO = 12? If so, what is the right angle? If not, why not?

3. A boy 2 meters tall casts a shadow 8 meters long. How long is a nearby flagpole if its shadow is 50 meters long?

4. Given right triangle PTU with altitude $\overline{TX}$.



   a. Name the similar triangles.

   b. If TX = 8, and PX = 4, what is UX?

Unit IV
Take-home Test

Name ____________________

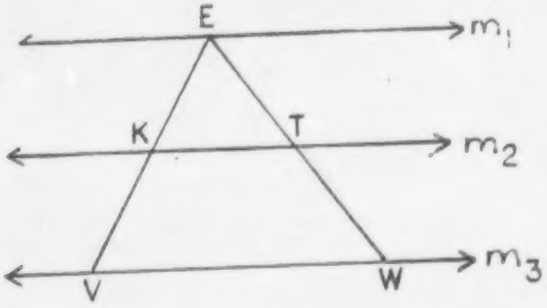
Draw a triangle and find the midpoint of each side. Join the midpoints with line segments.

How is each small triangle related to the triangle you first drew? How are the small triangles related to each other? Explain.

Unit IV
Review Worksheet

Name ______________________

1. On the right are parallel lines m_1, m_2, and m_3 cut by transversals to form triangles EKT and EVW. If EK = 4, EV = 16, KT = 7, EW = 20, and m ∠EKT = 65, then:

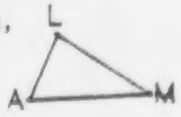


______ a. m ∠EVW = __?__

______ b. VW = __?__

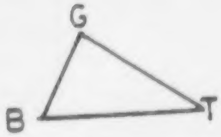
______ c. ET = __?__

2. Given Δ ALM ~ Δ BGT as shown.



______ a. If AL = 9 cm, LM = 12 cm, BG = 3 cm, and BT = 5 cm, find AM and GT.

______



______ b. If m ∠L = 45, and m∠B = 65, find m ∠G and m ∠M.

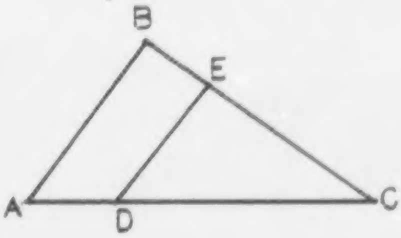
______

3. Complete the following using the Pythagorean Theorem.

______ a. Suppose ∠E of Δ REM is a right angle, RE = 6 and EM = 8, find RM.

______ b. Suppose ∠T of Δ BLT is a right angle, BT = 7 and BL = 25, find TL.

4. Suppose AB ∥ DE. If m∠BAC = 75, m ∠B = 60, AB = 20, DE = 15, DC = 18, and BC = 32. Then:



______ a. m ∠C = __?__

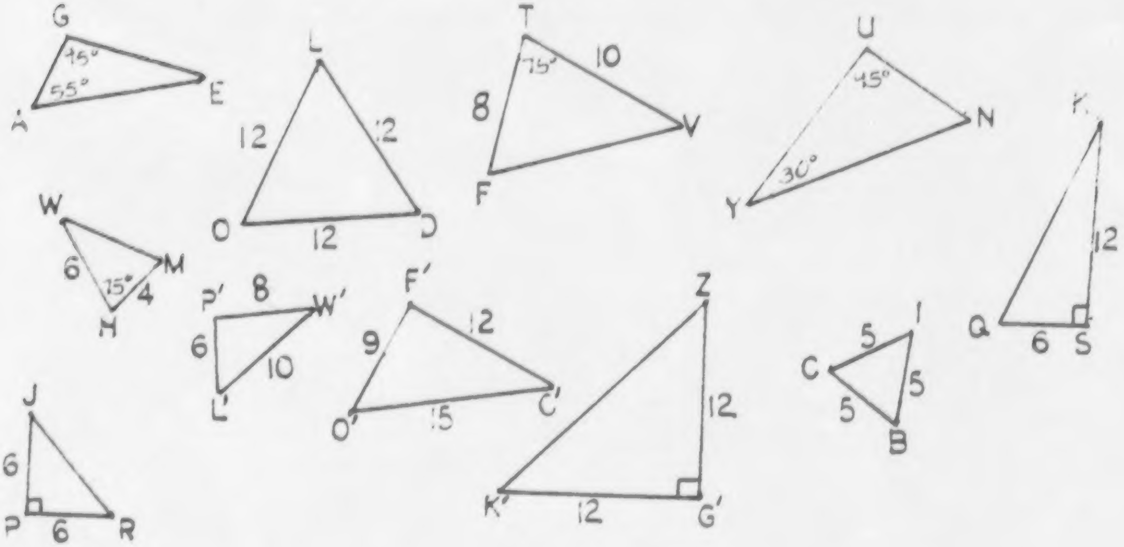
______ b. m ∠EDC = __?__

______ c. AC = __?__

______ d. EC = __?__

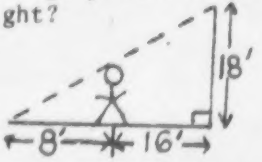
(Go on to the next page)

5. Name the pairs of similar triangles shown below. Indicate whether SAS, SSS, or AAA similarity shows that they are similar.


G
45°
55°
A
E
L
12
12
O
12
D
T
75°
10
8
V
F
U
45°
N
30°
Y
K
12
Q
6
S
W
6
75°
M
4
H
P'
8
W'
6
10
L'
F'
9
12
O'
15
C'
Z
12
K'
12
G'
C
5
I
5
5
B
J
6
P
6
R

______ 6. Is Δ MON a right triangle if MO = 4, MN = 5, and ON = 7? If so, what is the right angle? If not, why not?

______ 7. A person standing 16' from an 18' high pole casts an 8' shadow. What is the person's height?



______ 8. Suppose Δ CVW is similar to Δ KTU, m ∠ C = 55, m ∠ V = 85, m ∠ T = 85, m ∠ U = 40, what are the measures of the other angles?

______

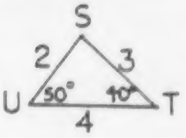
(Go on to the next page.)

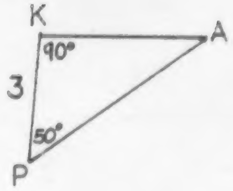
9. Complete each statement given the figures shown.

________ a. Δ STU ~ Δ __?__

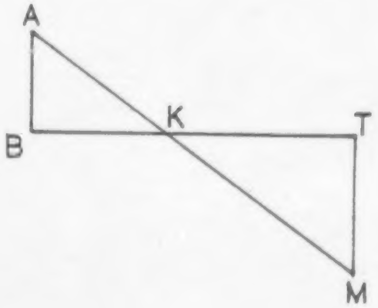
________ b. AK = __?__

________ c. AP = __?__





10. Suppose $\overline{AB} \perp \overline{BK}$ and $\overline{MT} \perp \overline{KT}$.
What can you say about Δ ABK and Δ MTK?
Explain.

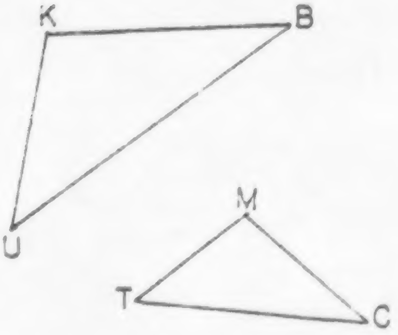


11. If, for Δ AGI, m ∠ G = 38, GI = 10, and AG = 12 and for Δ PRT, m ∠ R = 38, m ∠ P = 72, and RT = 5, is Δ AGI similar to Δ PRT?
Why or why not?

12. If, for Δ KQV, KQ = 15, KV = 10, m ∠ K = 40, and m ∠ Q = 30 and for Δ ENY, EN = 9, EY = 6, NY = 12, and m ∠ E = 40, is Δ KQV similar to Δ ENY?
Why or why not?

Unit IV Test Name ______________________

1. Given $\triangle BKU \sim \triangle CMT$ as shown.



________ a. If $m\angle B = 35$ and $m\angle T = 37$, find $m\angle U$ and $m\angle M$.
____

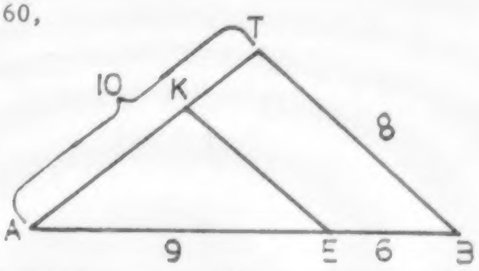
________ b. If $BK = 6$, $KU = 8$, $CM = 1.5$, and $CT = 2.5$, find BU and MT.
____

2. Complete the following using the Pythagorean Theorem.

________ a. Suppose $\angle P$ of $\triangle APW$ is a right angle, $AP = 5$, and $PW = 12$. Find AW.

________ b. Suppose $\angle T$ of $\triangle NRT$ is a right angle, $NT = 8$, and $NR = 17$. Find RT.

3. Suppose $\overline{EK} \parallel \overline{BT}$ as shown. If $AE = 9$, $BE = 6$, $BT = 8$, $AT = 10$, $m\angle AEK = 60$, and $m\angle ATB = 100$, then:



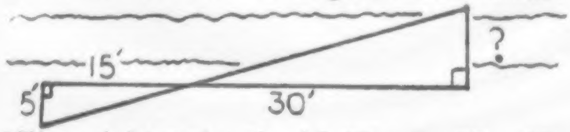
________ a. AK = ?

________ b. EK = ?

________ c. $m\angle ABT$ = ?

________ d. $m\angle A$ = ?

________ 4. Find the width of the river given the measures indicated.



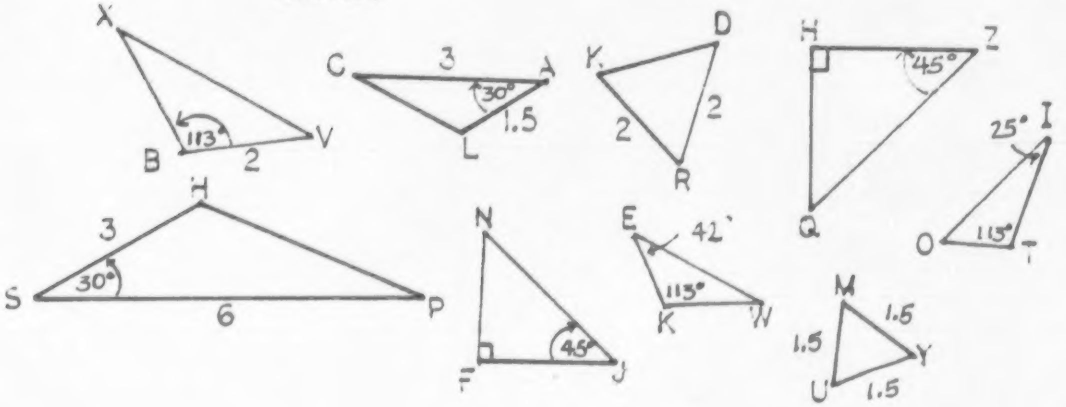
________ 5. Is $\triangle AQV$ a right triangle if $AQ = 9$, $AV = 12$, and $QV = 16$? If so what is the right angle? If not, why not?

6. If, for $\triangle BDF$, $m\angle D = 26$, $m\angle F = 100$, and $BD = 19$ and for $\triangle MKS$, $m\angle M = 54$, $m\angle K = 26$, and $SK = 5$, is $\triangle BDF$ similar to $\triangle MKS$? Why or why not?

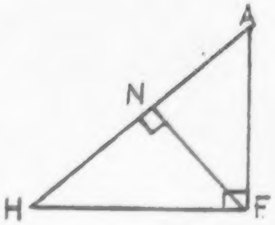
(Go on to the next page.)

__________ 7. What is the height of a tower whose shadow is 72 feet at the same time the shadow of a 6 foot person is 4 feet?

8. Name the pairs of similar triangles shown below. Indicate whether SAS, SSS, or AA Similarity shows that they are similar.


X
B
V
113°
2
C
A
L
3
30°
1.5
K
D
R
2
2
H
Z
Q
45°
25°
I
O
T
115°
H
S
P
3
30°
6
N
F
J
45°
E
K
W
42°
113°
M
Y
U
1.5
1.5
1.5

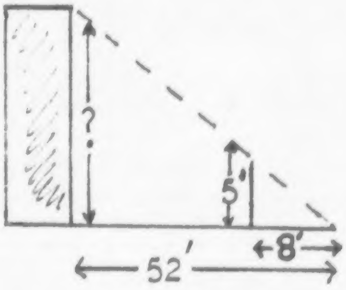
9. Given right triangle AFH with altitude $\overline{FN}$.



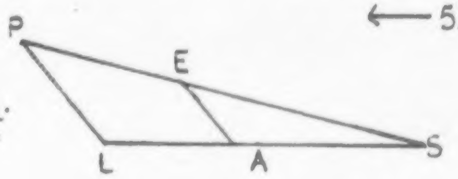
a. Name the similar triangles.

__________ b. If AN = 25, HN = 4, find FN.

__________ 10. Jan figured out a way to find the height of a building without using shadows. She lined up the top of the building and the top of a stick as shown. What is the height of the building?



11. Suppose $\overline{EA} \parallel \overline{PL}$.
Prove that $\frac{SE}{SP} = \frac{SA}{SL}$.



ANSWER KEY

NOTE: There may be more than one answer for some of the questions.

Quiz 1 *(10 points)*

(3) 1. a. $m\angle B = 60$, $m\angle C = 85$

b. $CP = 12.5$

(3) 2. a. $m\angle ARP = 50$ b. $AR = 15$ c. $PR = 21$

(4) 3. $\triangle ART \sim \triangle MQD$ by AAA Similarity

$\triangle BFG \sim \triangle LHU$ by AAA Similarity

$\triangle CIP \sim \triangle NOV$ by SSS Similarity

$\triangle KSX \sim \triangle YJE$ by SAS Similarity

Quiz 2 *(10 points)*

(2) 1. a. $JK = 10$ b. $LW = 16$

(2) 2. Yes, since the lengths of the three sides of the triangle are related so that the square of one is equal to the sum of the squares of the other two. The right angle is $\angle D$.

(2) 3. 12.5 meters

(4) 4. a. $\triangle PTX \sim \triangle PUT$ b. $UX = 16$

$\triangle TUX \sim \triangle PUT$

$\triangle PTX \sim \triangle TUX$

Take-home Test

Answers will vary.

Unit IV Review Worksheet

1. a. $m\angle EVW = 65$ b. $VW = 28$ c. $ET = 5$

2. a. $AM = 15$ cm, $GT = 4$ cm b. $m\angle G = 45$, $m\angle M = 70$

3. a. $RM = 10$ b. $TL = 24$

4. a. $m\angle C = 45$ c. $AC = 24$

b. $m\angle EDC = 75$ d. $EC = 24$

5. $\triangle AEG \sim \triangle NYU$ by AAA Similarity

$\triangle DOL \sim \triangle BCI$ by SSS Similarity ($\triangle DOL$ is also similar to $\triangle BIC$, $\triangle CBI$, $\triangle CIB$, $\triangle IBC$, and $\triangle ICB$)

$\triangle L'P'W' \sim \triangle O'F'C'$ by SSS Similarity

$\triangle JPR \sim \triangle K'G'Z'$ (or $\triangle JPR \sim \triangle ZGK'$) by SAS Similarity

(Go on to the next page.)

6. No, Δ MON is not a right triangle. The sides of the triangle are not related such that the square of the length of one side is equal to the sum of the squares of the lengths of the other sides.

7. 6'

8. m $\angle$ W = 40, m $\angle$ K = 55

9. a. Δ STU ~ Δ KAP

   b. AK = 4.5

   c. AP = 6

10. Answers may vary. Here is a possible proof:

    Δ ABK ~ Δ MTK. Since $\overline{AB} \perp \overline{BK}$ and $\overline{MT} \perp \overline{KT}$, it follows by the definition of perpendicular lines that $\angle$ ABK and $\angle$ MTK are right angles. Then $\angle$ ABK $\cong$ $\angle$ MTK. $\angle$ AKB and $\angle$ MKT are vertical angles by the definition of vertical angles. Then $\angle$ AKB $\cong$ $\angle$ MKT. Therefore, Δ ABK ~ Δ MTK by AA Similarity.

11. There is not enough information.

12. Yes, Δ KQV is similar to Δ ENY. Since $\frac{KQ}{KV} = \frac{EN}{EY}$ and m $\angle$ K = m $\angle$ E, it follows that the triangles are similar by the SAS Similarity.

Unit IV Test *(50 points)*

(8) 1. a. m $\angle$ U = 37, m $\angle$ M = 108

b. BU = 10, MT = 2

(6) 2. a. AW = 13 b. RT = 15

(8) 3. a. AK = 6 c. m $\angle$ ABT = 60

b. EK = 4.8 d. m $\angle$ A = 20

(2) 4. 10'

(3) 5. No, it is not a right triangle since the lengths of the three sides of the triangle are not related so that the square of the length of one side is equal to the sum of the squares of the lengths of the other two sides.

(4) 6. Yes, $\triangle BDF$ is similar to $\triangle MKS$. Since the sum of the measures of the angles in a triangle is 180 and $m\angle D = 26$ and $m\angle F = 100$, then $m\angle B = 54$. Since $m\angle K = 26$ and $m\angle M = 54$, it follows from the same reason that $m\angle S = 100$. Therefore $\triangle BDF$ is similar to $\triangle MKS$ by AAA Similarity.

(3) 7. 108'

(4) 8. $\triangle ACL \sim \triangle SPH$ by SAS Similarity
$\triangle DKR \sim \triangle MUY$ by SSS Similarity
($\triangle DKR$ is also similar to $\triangle MYU$, $\triangle YUM$, $\triangle YMU$, $\triangle UYM$, and $\triangle UMY$)
$\triangle HQZ \sim \triangle FJN$ (or $\triangle HQZ \sim \triangle FNJ$) by AA Similarity
$\triangle IOT \sim \triangle WEK$ by AA Similarity.

(3) 9. a. $\triangle FHN \sim \triangle AHF$, $\triangle AFN \sim \triangle AHF$, $\triangle FHN \sim \triangle AFN$

b. $FN = 10$

(3) 10. 32.5'

(4) 11. Answers may vary. Here is a possible answer:

Suppose $\overline{EA} \parallel \overline{PL}$. $\angle SEA$ and $\angle SPL$ and $\angle SAE$ and $\angle SLP$ are corresponding angles by the definition of corresponding angles. Then $\angle SEA \cong \angle SPL$ and $\angle SAE \cong \angle SLP$ by Theorem 14. By AA Similarity, it follows that $\triangle SEA \sim \triangle SPL$. Therefore, $\frac{SE}{SP} = \frac{SA}{SL}$ by the definition of similar triangles.

Unit V: Polygons and Polyhedra

Objectives

(1) To recognize and name polygons.

(2) To recognize trapezoids, parallelograms, rectangles, rhombuses, and squares.

(3) To classify triangles according to sides and according to angles.

(4) To recognize and name polyhedra.

(5) To investigate and determine the length of two sides of a 30-60 right triangle given the length of the hypotenuse or the length of the side opposite the 30-degree angle.

(6) To find the sum of the interior angles of a convex polygon.

(7) To find the number of lines through pairs of points, given a certain number of points, no three of which are collinear.

(8) To find the number of diagonals of a polygon.

Materials Needed

Set 1: scissors

Set 2: polyhedra models

Set 3: protractor, ruler

Set 7: packet III, tape

Suggestions for Teaching

Set 1, pages 138 - 141

Vocabulary: polygon; nonconvex; regular; trapezoid; rhombus; rectangle; acute, obtuse, and scalene triangles

Materials Needed: scissors

Comments: Be sure to have students discuss how they sorted the polygons in Exercise 1 on page 138. It is important for you to accept and encourage as many nonstandard ways of sorting as possible and not restrict their activities to the few suggested. For example, the figures may be sorted into the following categories: equilateral, equiangular, all acute angles, at least one right angle, at least two congruent sides, or 1 pair of parallel sides. The discussion should bring out terms that the students already know as well as some of the characteristics of the polygons.

PROBLEM SOLVING: SORTING FIGURES

Exercise 2c may be difficult for some students. Students should notice that the figures in each set have the same area. You may want to have them hold the shapes together to compare.

Have students suggest definitions for the new terms given the examples and the nonexamples. Note that the text defines a trapezoid as a quadrilateral with at least two sides parallel. Hence a parallelogram is a special case of a trapezoid.

It is not expected nor is it necessary for the students to come up with a formal definition for a polygon. A simple definition such as "a figure formed by line segments" with a picture is sufficient.

OCTA- PENTA- TRI-

Familiarize students with prefixes for geometric words because these prefixes give clues to the meaning of the words.

REAL WORLD EXAMPLES

Have students identify various polygons as you draw them. Include examples of polygons from the real world in these drawings. Also have students draw the different polygons, given their descriptions.

(Here are more problems similar to Exercises 6 - 8 and 13 - 15 on page 141 that you may want to assign:)

Is it possible to have the following figures? If so, draw one.

(1) An acute triangle that is isosceles
(2) An obtuse triangle that is isosceles
(3) A scalene triangle that is isosceles
(4) A right triangle that is equilateral
(5) An acute scalene triangle
(6) A trapezoid that is also a rhombus
(7) A parallelogram which is nonconvex
(8) A rhombus which is a rectangle

Exercises and activities like those above will help students see the relations of the special triangles to each other and the relations of the special quadrilaterals to each other. Another good exercise is to have students identify special quadrilaterals and special triangles from a given set of polygons. Supplementary Worksheet 1 provides exercises like these.

Set 2, pages 142 - 148

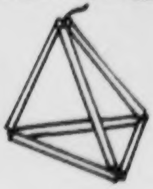
Vocabulary: polyhedra, prism, right prism, pyramid, right pyramid, bases, pentagonal prism, triangular prism, rectangular prism, cube, apex, rectangular pyramid, triangular pyramid, hexagonal pyramid, regular polyhedra

Materials Needed: polyhedra models

Comments: Many new terms are introduced in this set. Be sure to have students study the examples of the different types of polyhedra to see the properties they possess. Encourage them to use the new terms whenever possible.

NOTE: the special names for the regular polyhedra are not discussed at this time.

PHYSICAL MODELS



Provide practice in learning the new terms by having students identify drawings and physical models of the polyhedra.

There are many objects around us that can be used to illustrate the polyhedra. You or some of your students could also make straw or cardboard models to show the rest of the class. See Geometry and Visualization by the Mathematics Resource Project (University of Oregon, 1977) for suggestions of various methods and materials that can be used to make the models.

Directions for making the five regular polyhedra out of straw as well as how to stellate these polyhedra are given in Straw Polyhedra by Mary Laycock (Creative Publications, Inc., 1970).

Students should also practice drawing some of the simple polyhedra.

In the discussion of the different polyhedra, informally review or introduce the terms: edge, vertex, and face. For example, you may want to define a vertex as "the point at the corner of a polyhedron."

QUIZ I Anytime after Set 2

Set 3, pages 149 - 150

Vocabulary: 30-60 right triangle

Materials Needed: protractor, ruler

Comments: Students are asked to examine the sides of 30-60 right triangles to see the relationship between the hypotenuse and the side opposite the 30-degree angle. Have them discuss their findings for Exercises 1 through 3 before they do the rest of the exercises.

MAKE AND VERIFY CONJECTURES

Students may have to review Unit I, Set 5 in order to draw the 30-60 right triangles with specified lengths of hypotenuses.

Set 4, pages 151 - 152

Comments: Students must organize data, search for patterns, make conjectures, and test conjectures to find the sum of the measures of the interior angles of a convex polygon.

PROBLEM SOLVING

Suggest to the students that they draw the convex polygons with 4, 5, 6, and 7 sides; partition them into triangles by drawing all the diagonals from the one vertex (see polygon ABCDEFGH); and then complete the table.

LIMIT?

The term, "limit," used in Exercises 3 and 4 may be new to the students. Explain to them that it means "closest to a particular number." You may want to suggest an alternative question for Exercise 3: As the number of sides increase, is there a number that the sum of the measures of the angles gets closer and closer to? It would probably be best to illustrate with some number which someone suggests as a limit. For example, if 10,000 were suggested, then check n = 20, 30, 40, 50, and 60 (where n is the number of sides), and it should be obvious that the sum keeps getting larger and larger. Or if 360 were suggested for Exercise 4, then checking successive large number of sides would also show that the angle measure remains less than 180 but keeps getting close to it.

Set 5, pages 153 - 155

Vocabulary: Noncollinear points

Comments: For Exercise 13, have students guess what would happen, then check their guesses.

QUIZ 2

Anytime after Set 5

Set 6, pages 156 - 157

Comments: Students should be instructed to use the information they obtained in Set 5 to complete the second column in the table for Exercise 3.

PROBLEM SOLVING: LOOK AT ALTERNATIVES

In this set, students have the opportunity to work with and compare two different methods to find the number of diagonals in a convex polygon.

Set 7, pages 158 - 159

Vocabulary: regular polygons, regular polyhedron, cube, tetrahedron, octahedron, dodecahedron, icosahedron

Materials Needed: packet III, tape

Comments: Discuss the statement, "Every regular polygon can be inscribed in a circle," which is made on page 158.

PROBLEM SOLVING: MAKING MODELS

An alternative to having the students cut out the squares and triangles is to provide them with the figures already cut out in packet III. You may want to have the students work in pairs. Come together as a class to discuss the results.

Assign the Take-home Test. Students will need to use models of the polyhedrons. They may want to use the models they constructed in this set. If the students will not have a set of models to work with at home, they should be told to refer to drawings of polyhedrons like those on pages 142 - 143.

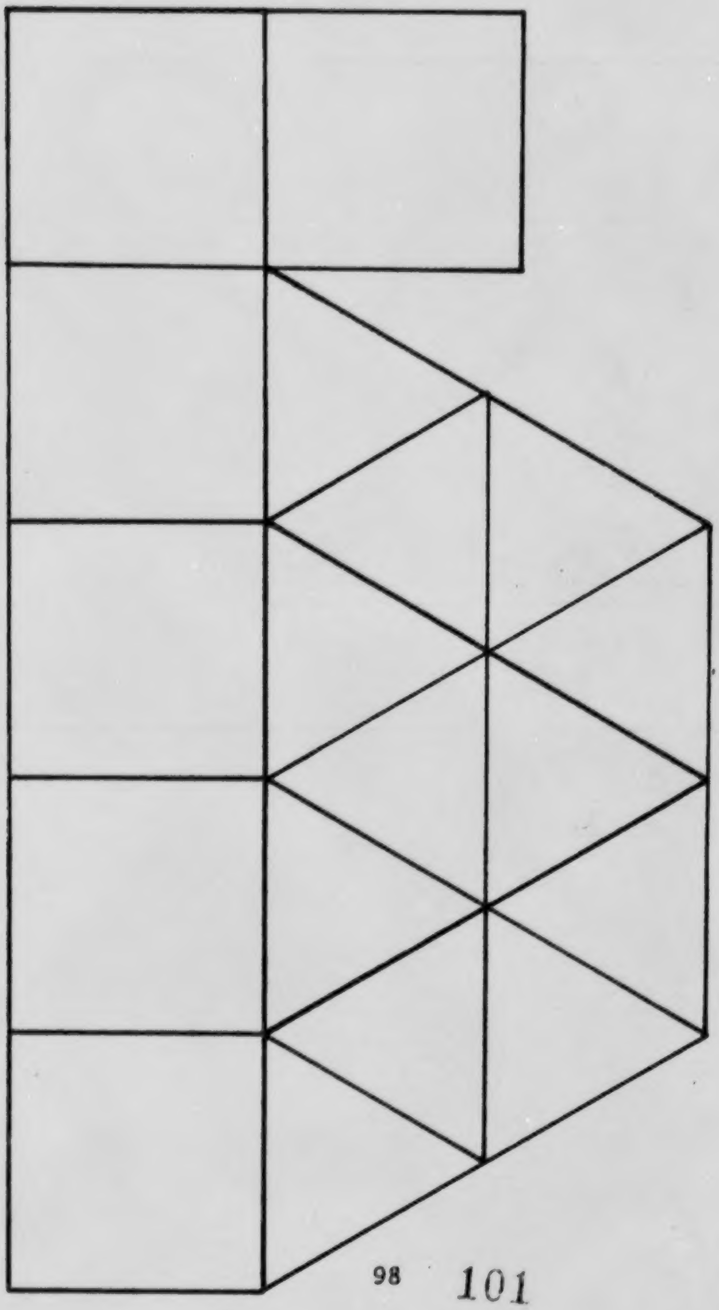
Review

A review worksheet is provided.

UNIT V TEST

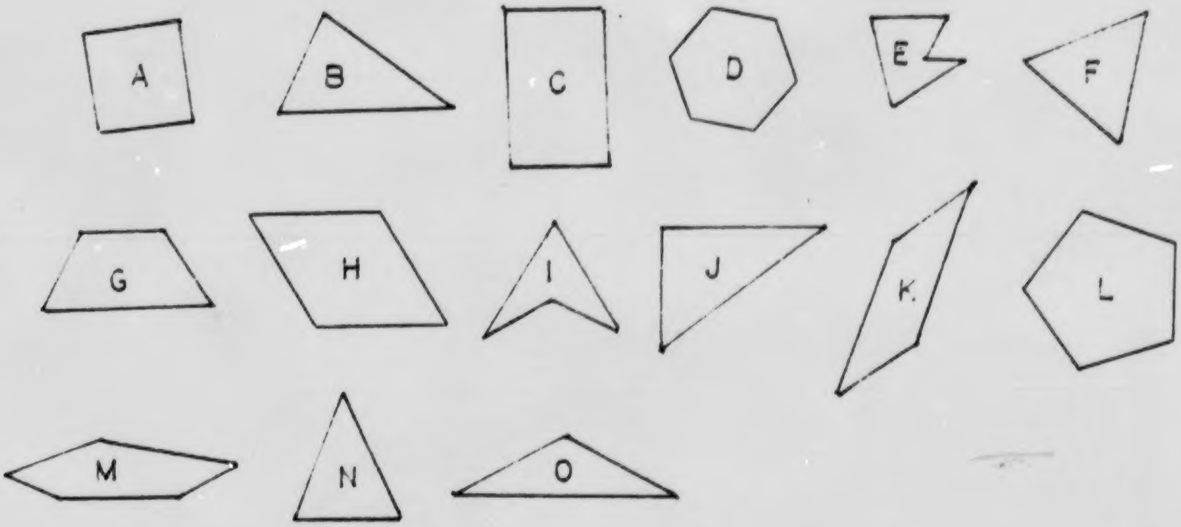
Unit V, Set 7
Packet III

Directions:
1) Make a ditto master of this sheet.
2) Ditto copies on 5" by 8" index cards.
3) Cut out the squares and the triangles. Place figures in envelopes.



Unit V
Supplementary Worksheet 1

Name ______________________________



List all the figures that appear to be:

__________ 1. rectangles

__________ 2. acute triangles

__________ 3. trapezoids

__________ 4. parallelograms

__________ 5. pentagons

__________ 6. regular polygons

__________ 7. nonconvex polygons

__________ 8. rhombuses

__________ 9. isosceles triangles

__________ 10. obtuse triangles

Caution: There may be more than one answer for each problem!

Unit V, Sets 4 - 6 Name ____________________

Supplementary Worksheet 2

1. For each of the regular polygons below:

   1) Find the sum of the measures of the interior angles.
   2) Find the measure of one interior angle.

__________ a. heptagon __________ b. 15-gon __________ c. 20-gon

__________ __________ __________

________ 2. What is the measure of each angle in a stop sign?

3. Given each of the following number of points, with no three collinear, how many lines can you draw through pairs of points?

__________ a. 8 points __________ c. 30 points

__________ b. 14 points __________ d. 400 points

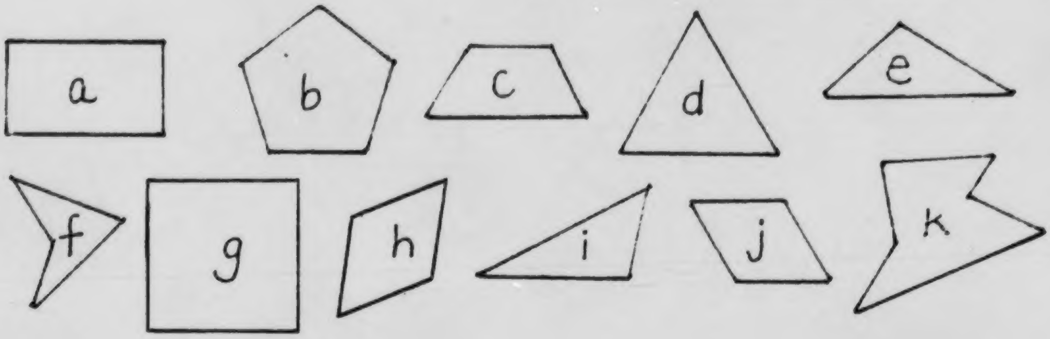
4. Find the number of diagonals for each of the following polygons?

   a. An octagon

   b. a 20-sided polygon

Unit V
Quiz I

Name____________________________



List all the figures above that appear to be:
(There may be more than one answer for each problem!)

___________ 1. trapezoids

___________ 2. parallelograms

___________ 3. rhombuses

___________ 4. rectangles

___________ 5. nonconvex polygons

___________ 6. regular polygons

___________ 7. scalene triangles

___________ 8. acute triangles

___________ 9. obtuse triangles

List all the figures below that appear to be:
(There may be more than one answer for each problem!)

___________ 10. prisms

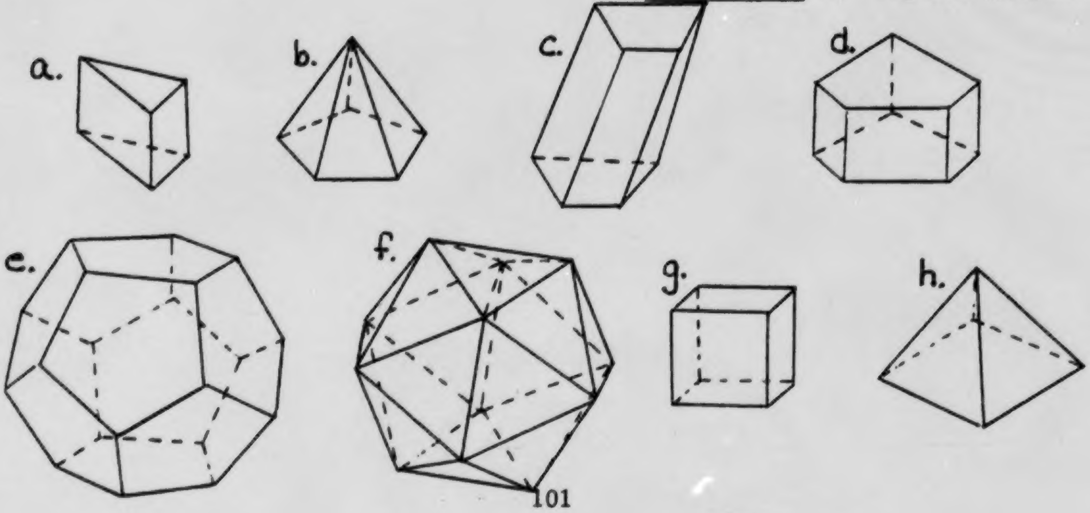
___________ 11.. right prisms

___________ 12. pentagonal prisms

___________ 13. pyramids

___________ 14. rectangular pyramids

___________ 15. regular polyhedra



Unit V
Quiz 2

Name ______________________________

1. Suppose the hypotenuse of a 30-60 right triangle were 20 centimeters long. Find the lengths of the other two sides of the triangle.

2. Suppose the side opposite the 30-degree angle of a 30-60 right triangle were 4 centimeters long. Find the length of the other two sides of the triangle.

3. Find the sum of the measures of the interior angles of a convex hexagon.

4. Find the measure of one angle of a regular decagon.

5. Given 8 points, no three collinear, how many lines can you draw through pairs of points?

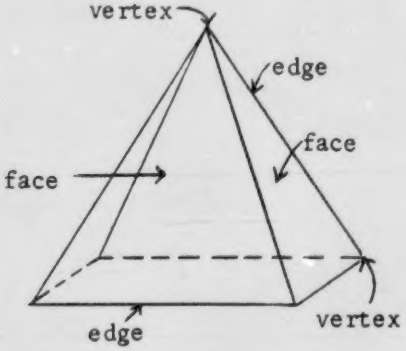
Unit V

Name ______________________________

Take-home Test

Using models of polyhedrons, count the number of vertices, faces, and edges, and complete the table below.

The pictured rectangular-pyramid has 5 faces, 5 vertices, and 8 edges.



Polyhedron	No. of Vertices V	No. of Faces F	No. of Edges E	V + F
__________				
__________				
__________				
__________				
__________				
__________				

What is the relationship between V + F and E? Can you find a polyhedron which does not satisfy this relationship?

Unit V
Review Worksheet

Name ____________________________

Show all your work on a separate sheet of paper.

1. Define each of the following terms and draw a picture of each:
    - a. nonconvex polygon
    - b. regular polygon
    - c. quadrilateral
    - d. trapezoid
    - e. parallelogram
    - f. rectangle
    - g. square
    - h. rhombus
    - i. right triangle
    - j. acute triangle
    - k. obtuse triangle
    - l. scalene triangle
    - m. isosceles triangle
    - n. equilateral triangle
    - o. prism
    - p. right prism
    - q. pyramid
    - r. right pyramid

2. Define each of the following terms:
    - a. regular polyhedron
    - b. cube
    - c. octahedron
    - d. tetrahedron
    - e. icosohedron
    - f. dodecahedron

3. Identify each of the polygons that appear on page 139 in your book.

4. Identify each of the polyhedra that appear on pages 142 - 143 in your book.

5. Suppose the side opposite the 30-degree angle of a 30-60 right triangle were 10 decimeters long. Find the lengths of the other two sides of the triangle.

6. Suppose the hypotenuse of a 30-60 right triangle were 16 centimeters long. Find the lengths of the other two sides.

7. For each of the regular polygons below:
    - 1) Find the sum of the measures of the interior angles.
    - 2) Find the measure of one interior angle.

    a. nonagon b. decagon c. n-gon

8. Given each of the following number of points, with no three collinear, how many lines can you draw through pairs of points?

    a. 5 points b. 12 points

9. Find the number of diagonals for each of the following polygons:

    a. heptagon b. decagon

Unit V Test   Name ____________________________

Show all your work on a separate sheet of paper. Be sure to use a straightedge for all the drawings!

1. List all the figures that appear to be:

   a. acute triangles

   b. obtuse triangles

   c. quadrilaterals

   d. trapezoids

   e. parallelograms

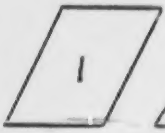
   f. rectangles

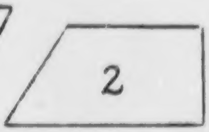
   g. rhombuses

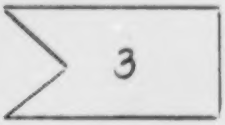
   h. squares

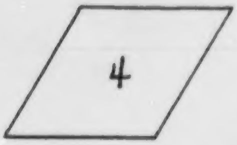
   i. regular polygons

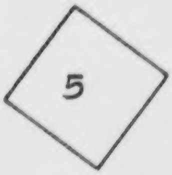
   j. nonconvex polygons

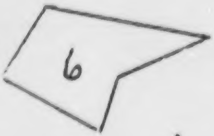


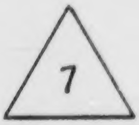


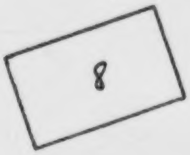


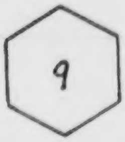


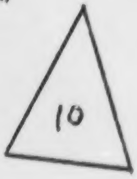


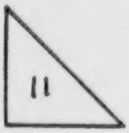


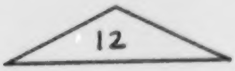


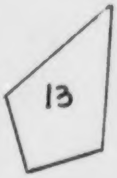


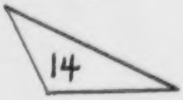








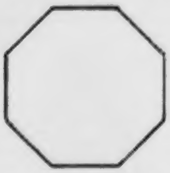




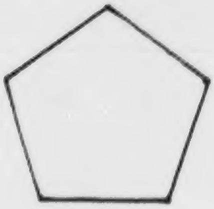
2. Each of the polygons below are regular polygons.

   a. Name the polygon.
   b. Find the sum of the measures of the interior angles.
   c. Find the measure of one interior angle.

1)



2)



3. Draw each of the following figures, if possible. If there is no such figure, write NONE.

   a. an isosceles, acute triangle

   b. a cube

   c. a triangular pyramid

   d. an isosceles, scalene triangle

   e. a rectangle which is a rhombus

(Go on to the next page.)

4. Tell whether each statement is true or false.

   a. Every trapezoid is a parallelogram.

   b. No right triangles are scalene.

   c. Some isosceles triangles are obtuse.

   d. Every quadrilateral that has all sides congruent is a square.

   e. A regular dodecahedron is a regular polyhedron with pentagonal faces.

5. Find the number of diagonals for each of the following polygons. Be sure to show all your computations and/or drawings.

   a. a hexagon

   b. an octagon

6. Given each of the following number of points, with no three collinear, how many lines can you draw through pairs of points?

   a. 6 points

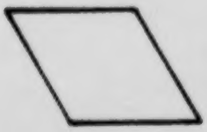
   b. 10 points

7. Suppose the hypotenuse of a 30-60 right triangle were 24 centimeters long. Find the lengths of the other two sides of the triangle.

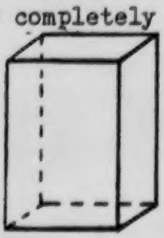
8. Suppose the side opposite the 30-degree angle of a 30-60 right triangle were 2 centimeters long. Find the lengths of the other two sides of the triangle.

9. Identify each of the following as completely as you can.

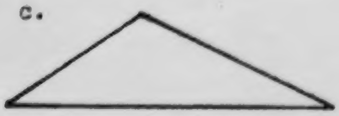
a.



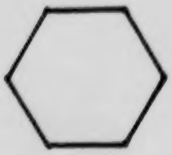
b.



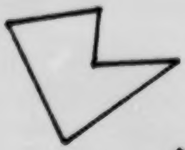
c.



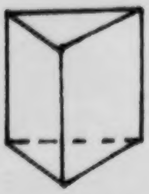
d.



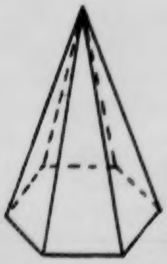
e.



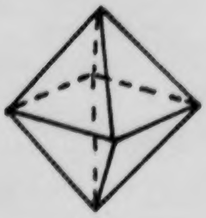
f.



g.



h.



ANSWER KEY

Supplementary Worksheet 1

1. A, C
2. B, F, N
3. A, C, G, H, K
4. A, C, H, K
5. E, L
6. A, D, F, L
7. E, I
8. A, H
9. F, N
10. O

Supplementary Worksheet 2

1. a. 1) 900°   b. 1) 2340°   c. 1) 3240°
   2) 128 4/7°   2) 156°   2) 162°

2. 135°

3. a. 28 lines   c. 435 lines

   b. 91 lines   d. 79,800 lines

4. a. 20 diagonals   b. 170 diagonals

(1 point each)

Quiz 1 *(15 points)*

1. a, c, g, h, j
2. a, g, h, j
3. g, h, j
4. a, g
5. f, k
6. b, d, g
7. e, i
8. d
9. e, i
10. a, c, d, g
11. a, d, g
12. d
13. b, h
14. h
15. e, f

Quiz 2 *(10 points)*

(2) 1. The side opposite the 30° angle is 10 centimeters long and the side opposite the 60° angle is $\sqrt{300}$ centimeters long or $10\sqrt{3}$ centimeters long.

(2) 2. The hypotenuse is 8 centimeters long and the side opposite the 60° angle is $\sqrt{48}$ centimeters long or $4\sqrt{3}$ centimeters long.

(2 points each) 3. 720°   4. 144°   5. 28 lines

(Go on to the next page.)

Take-home Test

Answers will vary for the table, depending on the polyhedrons selected. Here are the answers for the regular polyhedra:

POLYHEDRON	NO. of VERTICES	NO. of FACES	NO. of EDGES	V + F
cube	8	6	12	14
tetrahedron	4	4	6	8
octahedron	6	8	12	14
dodecahedron	20	12	30	32
icosahedron	12	20	30	32

The relationship between V + F and E is V + F = E + 2. All polyhedra satisfy the relationship.

Review Worksheet

1. Answers may vary.

2. Answers may vary.

3. Answers may vary. Here are possible answers:

FIGURE	NAME
a	a parallelogram
b	an equilateral triangle or an acute triangle
c	an isosceles triangle or an acute triangle
d	a rectangle
e	a regular pentagon
f	a convex pentagon
g	a nonconvex pentagon
h	a rectangle
i	a nonconvex quadrilateral
j	a scalene, obtuse triangle
k	a regular hexagon
l	an acute, scalene triangle
m	an acute, scalene triangle
n	a rhombus
o	a parallelogram
p	a right, scalene triangle
q	a trapezoid
r	a regular hexagon
s	a nonconvex hexagon
t	a nonconvex hexagon
u	a square
v	a convex pentagon
w	an acute, scalene triangle
x	a right, scalene triangle

(Go on to the next page.)

4.

FIGURE	NAME
a	a rectangular pyramid
b	a pentagonal, right prism
c	a triangular prism
d	a triangular, right prism
e	an octahedron
f	a hexagonal, right pyramid
g	a rectangular, right prism
h	a rectangular, right prism or a cube
i	a triangular, right pyramid or a tetrahedron
j	a dodecahedron
k	a pentagonal prism
l	a rectangular prism
m	a triangular prism
n	a hexagonal pyramid
o	an icosahedron

5. The hypotenuse is 20 decimeters long. The side opposite the 60-degree angle is $\sqrt{300}$ decimeters or $10\sqrt{3}$ decimeters long.

6. The side opposite the 30-degree angle is 8 cm and the side opposite the 60-degree angle is $\sqrt{192}$ or $8\sqrt{3}$ centimeters long.

7. a. 1) 1260° b. 1) 1440° c. 1) 180(n - 2) degrees
   2) 140° 2) 144° 2) $\frac{180(n - 2)}{n}$ degrees

8. a. 10 lines b. 66 lines

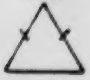
9. a. 14 diagonals b. 35 diagonals

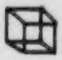
Unit V Test *(50 points)*

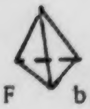
(10) 1. a. 7, 10 b. 12, 14 c. 1, 2, 4, 5, 8, 13
d. 1, 2, 4, 5, 8 e. 1, 4, 5, 8 f. 5, 8
g. 4, 5 h. 5 i. 5, 7, 9
j. 3, 6

(10) 2. 1. a. octagon 2. a. pentagon
b. 1080° b. 540°
c. 135° c. 108°

(5) 3. Drawings may vary. Examples are shown.

a. 

b. 

c. 

d. none

e.

(5) 4. a. F b. F c. T d. F e. T

(4) 5. a. 9 diagonals b. 20 diagonals

(4) 6. a. 15 lines b. 45 lines

(2) 7. The side opposite the 30° angles is 12 centimeters and the side opposite the 60° angle is $\sqrt{432}$ (or $12\sqrt{3}$) centimeters.

(2) 8. The hypotenuse is 4 centimeters long and the side opposite the 60° angle is $\sqrt{12}$ (or $2\sqrt{3}$) centimeters.

(8) 9. Answers may vary.

a. rhombus
b. rectangular prism
c. obtuse, scalene triangle
d. regular hexagon
e. nonconvex pentagon
f. right, triangular prism
g. hexagonal pyramid
h. octahedron

Unit VI: Circles and Sphere

Objectives

(1) To identify points, segments, and lines related to circles.

(2) To identify major and minor arcs and semicircles.

(3) To identify and find the degree measures of central angles of circles.

(4) To find the degree measures of arcs of circles.

(5) To investigate and use the formula for the circumference of a circle.

(6) To find the length of an arc of a circle.

(7) To identify and find the degree measures of inscribed angles of circles.

(8) To identify points, segments, lines, and circles related to a sphere.

Materials Needed

Set 3: can of soup or juice, quarter, other circular objects, and tape measure or string

Set 5: physical model of a sphere

Suggestions for Teaching

Set 1, pages 162 - 164

Vocabulary:	circle, center, radius, chord, diameter, tangent, point of tangency, secants
Comments:	Help students define the new terms based on the examples provided. Most of these terms should already be familiar to the students.

Oral drills on vocabulary will ease the learning process for the students. You might draw a circle and then add a radius, a chord, and so on, one at a time. Also, have students draw the segments and lines of the circle as their special names are stated.

Remind students that we use the terms, radius and diameter, to denote both the segments and the segment lengths.

You may want to have the students do the activity suggested below:

(1) Draw a circle on a sheet of paper. Mark the center, and a point A on the circle.
(2) Fold the sheet along the diameter through point A.
(3) Then fold again through A so that part of the first fold falls on itself.
(4) Why is the second fold a tangent to the circle?

Set 2, pages 165 - 166

Vocabulary: arc, minor arc, major arc, semicircles

Symbols: $\overset{\frown}{AB}$, $\overset{\frown}{ADB}$

Comments: Help students to define and use the new terms. They should be provided with practice in identifying the different types of arcs of the circle.

QUIZ 1 Anytime after Set 2

Set 3, pages 167 - 171

Vocabulary: circumference, π(pi), central angle, intercepted by, concentric circle

Symbols: m $\overset{\frown}{AB}$

Materials Needed: can of soup of juice, quarter, other circular objects, and tape measure or string

Comments: If a tape measure is not available for Exercise 1 on page 167, students may use a piece of paper or string to measure the circular objects, then measure the paper or string with the ruler. Students should be encouraged to eyeball the circumference to check their answers for reasonableness.

PROBLEM-SOLVING: STUDY TABLES In Exercise 1, students are asked to study their tables to see if they notice any patterns in the ratios of the circumference to diameter. Have students discuss their conjectures and suggest they check them by measuring the circumferences and diameters of a few more objects.

WRITE FORMULAS Allow time for students to write the formulas asked for in Exercise 2. Then discuss the formulas and a few examples as a class, before assigning Exercises 3 - 6.

π
"PI"

A discussion on when 3.14 or 22/7 are preferable to use as an approximation of π is recommended after students have completed Exercise 3 on page 168.

For Exercises 4 and 5, you may want to have students give their answers with "π" before using either 3.14 or 22/7 as an approximation. Students should realize that they can write their answers as 4π rather than 12.56 or 88/7.

If a basketball is not available for students to do Exercise 6, have them find the diameter of a softball, volleyball, and so on.

CON-
-CENTRIC

Point out to the students that the prefix *"con-"* and the stem *"-centric"* mean "together" and "center," respectively.

In Exercises 14 and 15 on page 170, students are again asked to study the tables they have completed for patterns. Have students discuss their results as a class. Then do Exercises 15 and 16 as a class.

A supplementary worksheet containing more exercises on finding the circumference of a circle and the length of an arc is provided.

Set 4, pages 172 - 174

Vocabulary: inscribed, circumscribed

Comments: Exercise 2b may be difficult for some students. You may have to provide hints such as draw radii $\overline{AO}$ and $\overline{CO}$. Discuss exercises 1, 2a, and 2b as a class before assigning the rest of the exercises in Set 4.

Be sure to have students explain how they obtained their answers for the exercises. This will help them to distinguish between an inscribed angle and a central angle.

Transparency masters of pages 172 - 174 are provided to help you in discussing the problems in class.

QUIZ 2 Anytime after Set 4

Set 5, page 175

Vocabulary: sphere, great circle

Comments: A physical model of a sphere should be shown.

Set 6, pages 176 - 178

Comments: Students must use conclusions from the previous set as well as from some earlier units to do the exercises in this set.

Be sure to have students explain how they obtained their answers.

Transparency masters of pages 176 - 178 are provided.

Assign Take-home Test.

Review

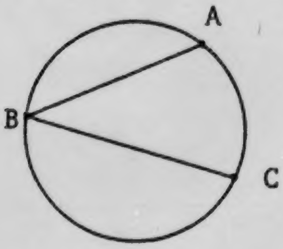
A review worksheet for this unit is provided.

UNIT VI TEST

Transparency 1
Unit VI Circles and Spheres

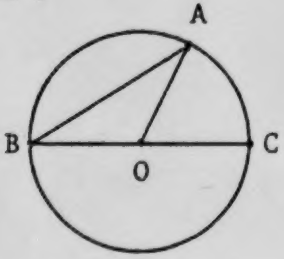
Set 4

An inscribed angle is another angle associated with a circle.



If A, B, and C are points of the circle, $\angle ABC$ is an inscribed angle.

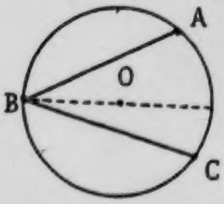
1. Using the special inscribed angle shown below, find its degree measure in terms of $\widehat{AC}^\circ$.



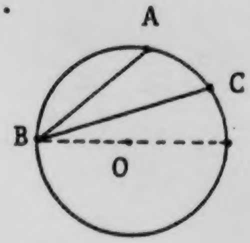
O is the center of the circle. (Hint: Since $\angle AOC$ is a central angle, what is its degree measure? How is $\angle AOC$ related to $\triangle ABO$? How does m $\angle AOC$ compare with m $\angle ABO$?)

2. The idea from Exercise 1 can be extended to an arbitrary inscribed angle. Show that in each case below, m $\angle ABC = \frac{1}{2}\widehat{AC}^\circ$.

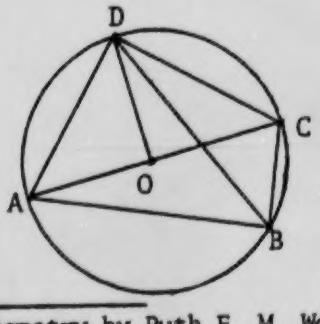
a.



b.



3. If O is the center of the circle, $\widehat{BC}^\circ = 50$ and $\overline{DO} \perp \overline{AC}$, find the following:



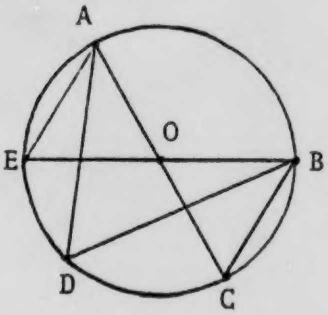
a. m $\angle BAC$
b. m $\angle BDC$
c. $\widehat{AB}^\circ$
d. m $\angle ACB$
e. $\widehat{DC}^\circ$
f. m $\angle ABD$
g. m $\angle AOD$
h. m $\angle ACD$

From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 172.

118

Transparency 2
Unit VI Set 4

4. If O is the center of the circle, $\widehat{AB}^\circ = 120$, find the following:

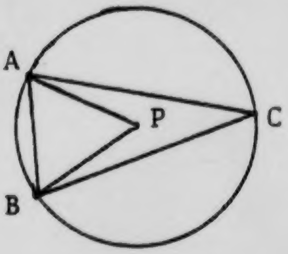


a. m ∠AEB
b. m ∠ADB
c. m ∠ACB
d. m ∠AOB
e. m ∠AOE

What conclusion can you draw from a, b, c?

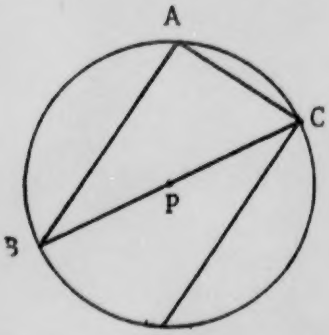
5. Draw a circle with center O. Let $\overline{PQ}$ be a diameter. Choose three points, A, B, C, on the same semicircle $\widehat{PQ}$. ∠PAQ, ∠PBQ, and ∠PCQ are inscribed in semicircle $\widehat{PQ}$. Measure these angles. What can you say about an angle inscribed in a semicircle?

6. If P is the center of the pictured circle and $\widehat{AB}^\circ = 60$, find the following



a. m ∠APB
b. m ∠ACB
c. m ∠PAB

7. P is the center of the pictured circle, $\overline{AB} \parallel \overline{CD}$, and m ∠ABC = 40. Find

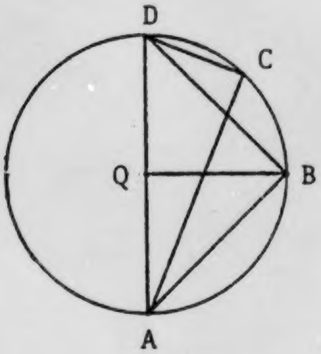


a. $\widehat{AC}^\circ$
b. m ∠BCD
c. $\widehat{BD}^\circ$
d. m ∠BAC
e. m ∠ACB

119

From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 173.

8. If Q is the center of the pictured circle, m ∠BQD = 90, and ∠BAC ≅ ∠CAD, find



a. m ∠ABD

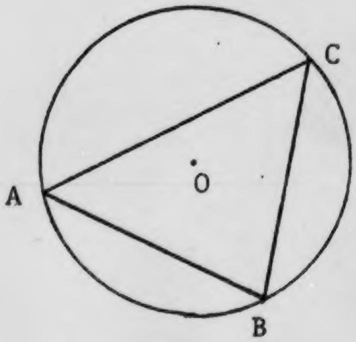
b. m ∠ACD

c. $\overset{\frown}{AB}°$

d. $\overset{\frown}{BC}°$

e. $\overset{\frown}{CD}°$

9.



ΔABC which has its vertices on a circle is said to be inscribed in the circle and the circle is circumscribed about the triangle.

Draw some triangles and see if you can inscribe them in circles. (You may find it helpful to go back to Set 5 of Unit I, Congruence, on perpendicular bisectors of segments to find the centers of your circles.)

Do you think every triangle can be circumscribed in some circle?

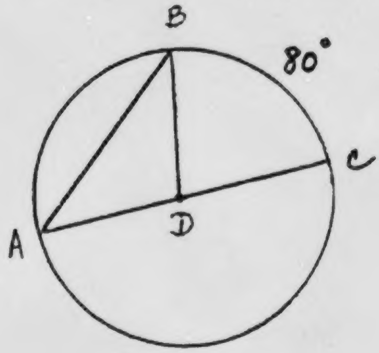
From *Geometry* by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 174.

Transparency 4
Unit VI Circles and Spheres

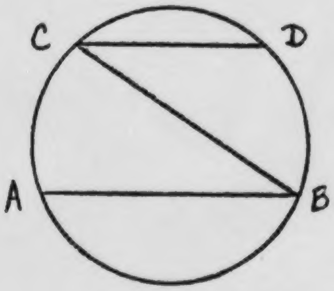
Set 6

Do the following exercises, using conclusions from the preceding sets as well as some from earlier units.

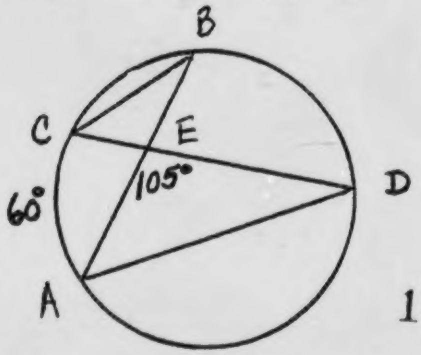
1. If $\overline{AC}$ is a diameter of a circle with center D, find m ∠ BDC and m ∠ BAC.



2. If $\widehat{AC}^\circ = \widehat{BD}^\circ$, what do you know about ∠ DCB and ∠ ABC? Why? Then what can you say about $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$?



3. If m ∠ AED = 105° and $\widehat{AC}^\circ = 60$, find

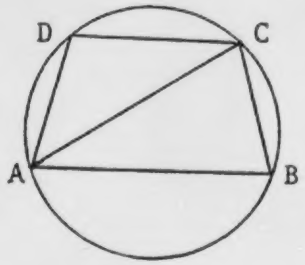


a. m ∠ ABC
b. m ∠ ADC
c. m ∠ BEC
d. m ∠ BCD
e. $\widehat{BD}^\circ$

From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 176.

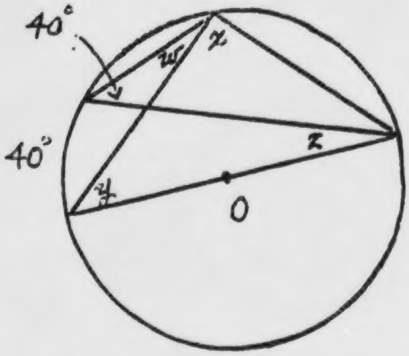
Transparency 5
Unit VI   Set 6

4. If $\widehat{AD}^\circ = \widehat{BC}^\circ$ in the picture below, what kind of figure is ABCD?

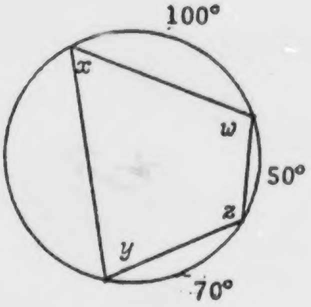


Determine the values of x, y, z, and w below.

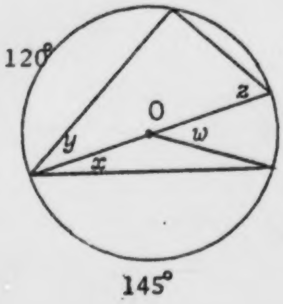
5.



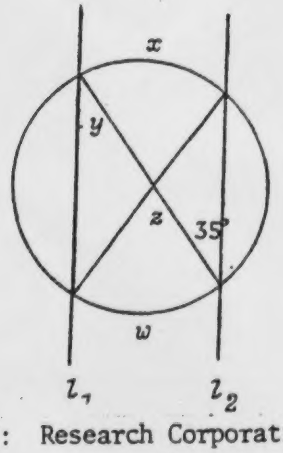
6.


100°
x
w
50°
z
y
70°

7.


120°
O
z
w
y
x
145°

8.


x
y
z
35°
w
l_1
l_2

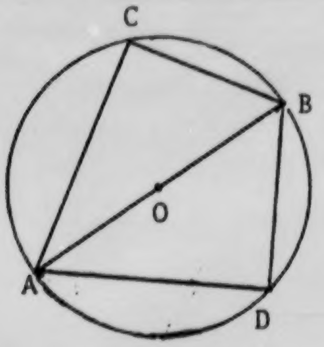
$l_1 \parallel l_2$

122

From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 177.

Transparency 6
Unit VI Set 6

9. If $\overline{AB}$ is a diameter of a circle with center O, $\widehat{AC}^\circ = 120$ and $\widehat{AD}^\circ = 55$, find



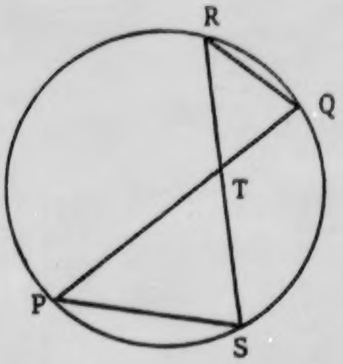
a. m ∠BAC

b. m ∠BAD

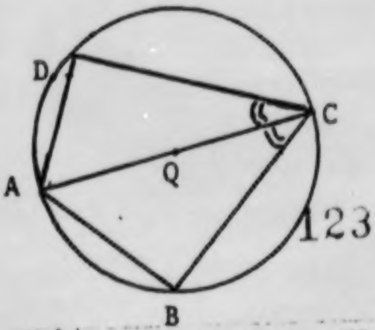
c. m ∠CBD

d. m ∠ACB

10. If P, Q, R, S are points on a circle and $\overline{PQ}$ and $\overline{RS}$ intersect at T, how are ΔPST and ΔRQT related? Explain.



11. If Q is the center of the circle below, A, B, C, D are points of the circle, and ∠DCA ≅ ∠BCA, do you have enough information to say that $\Delta ABC \cong \Delta ADC$? If so, how do you know the triangles are congruent? If not, what other information do you need?



From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 178.

Unit VI, Set 3
Supplementary Worksheet 1

Name ______________________

1. Find the circumference of the circle in terms of π.

______ a. diameter = 3 cm ______ c. radius = 5 dm

______ b. radius = 12 m ______ d. diameter = 40 cm

2. The numbers 3.14 and 22/7 are common approximations of π. Use one of these to find the circumference of a circle whose diameter is

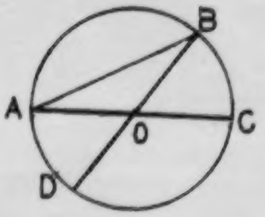
______ a. 28 cm ______ b. 60 cm

3. What is the circumference of a circle whose radius is 15 m? Use $\pi = 3.14$ or 22/7.

4. If $\overline{AC}$ and $\overline{BD}$ are diameters of the circle with center O, and $\overset{\frown}{DC} = 130$, give the degree measure of

______ a. $\overset{\frown}{BC}$ ______ c. $\overset{\frown}{AB}$

______ b. $\overset{\frown}{ABC}$ ______ d. $\overset{\frown}{ACD}$



5. Find the lengths of the arcs if

______ a. the circle has radius 4 m and the arc has degree measure 40.

______ b. the circle has diameter 20 mm and the arc has degree measure 36.

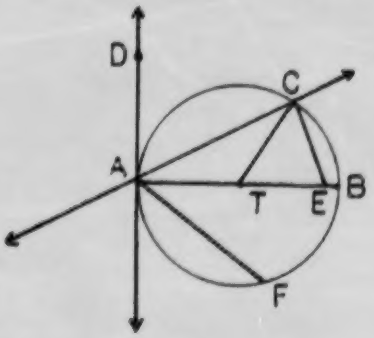
______ c. the circle has circumference 20 cm and the arc has degree measure 90.

______ d. the circle has diameter 33 cm and the arc has degree measure 120.

Unit VI
Quiz 1

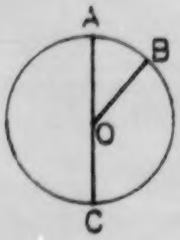
Name ____________________

Identify each of the following using the circle with center T in the figure on the right.



1. $\overline{AB}$ ____________________
2. $\overline{AF}$ ____________________
3. $\overline{CT}$ ____________________
4. $\overleftrightarrow{AD}$ ____________________
5. $\overleftrightarrow{AC}$ ____________________

Refer to circle with center O in the diagram shown.



6. Name a minor arc. ____________________
7. Name a major arc. ____________________
8. Name a semicircle. ____________________

Define each of the following terms and draw a picture of each:

9. a chord of a circle

10. a minor arc

Unit VI
Quiz 2

Name ____________________

1. Use either $\pi = 3.14$ or $\pi = \frac{22}{7}$ to find the circumference of a circle whose diameter is

____________ a. 28 centimeters

____________ b. 5 meters

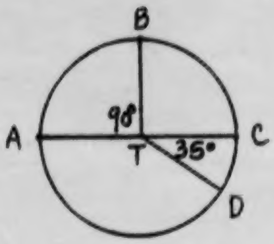
________ 2. What is the circumference of a circle with radius 4 cm? (Use $\pi = 3.14$ or $\pi = 22/7$.)

3. Find the degree measure of each of the following arcs in circle T.

____________ a. $\overset{\frown}{AB}$

____________ b. $\overset{\frown}{AD}$

____________ c. $\overset{\frown}{BCD}$



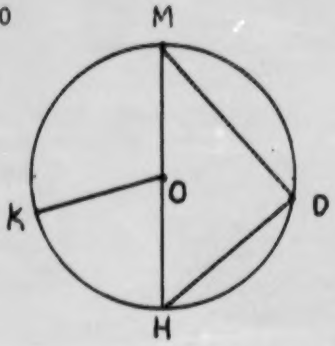
__________ 4. Find the length of an arc if the circle has circumference 27 cm and the arc has degree measure 40 .

5. If O is the center of the circle, $\overset{\frown}{MD}° = 100$ and $\overset{\frown}{KM}° = 100$, find the following:

____________ a. m∠ MHD

____________ b. m∠ DMH

____________ c. m∠ KOH



Unit VI Name_______________________________

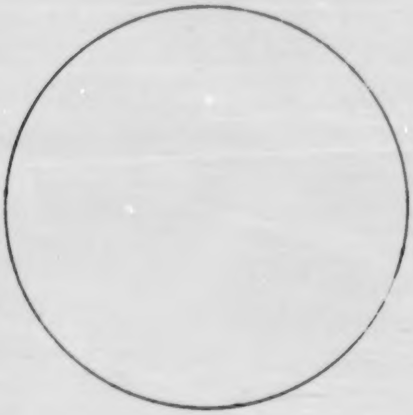
Take-home Test

Suppose you had a circle whose center was not marked. How can you find the center? (There is more than one way to answer this problem.)

Laurie suggested the following procedure:

> Draw any two chords of the circle which are not parallel.
> Then construct their perpendicular bisector.

Laurie claims that the point of intersection of the perpendicular bisector is the center of the circle. Follow Laurie's suggestion for the circle given below. Do you agree?



Unit VI
Review Worksheet

Name ______________________

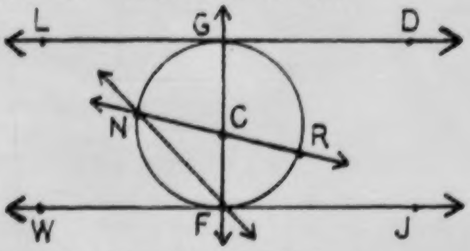
Show all your written work on a separate sheet of paper.

1. Define and draw a picture of each of the following terms:

   a. a circle
   b. a radius of a circle
   c. a diameter of a circle
   d. a chord of a circle
   e. a secant of a circle
   f. a tangent of a circle
   g. point of tangency
   h. an arc of a circle
   i. a minor arc
   j. a major arc
   k. semicircle
   l. circumference of a circle
   m. a central angle of a circle
   n. concentric circles
   o. an inscribed angle
   p. a sphere
   q. a great circle

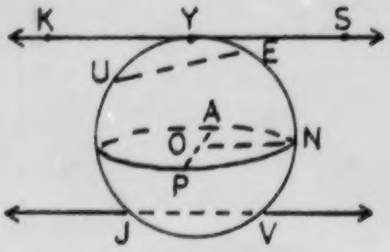
2. Refer to the diagram of the circle with center C. Name each of the following:

   a. 3 radii
   b. a diameter
   c. a secant
   d. a tangent
   e. a chord that is not a diameter
   f. a point of tangency



3. Refer to the diagram of the sphere with center O. Name each of the following:

   a. 4 points on the sphere
   b. 3 radii of the sphere
   c. a diameter of the sphere
   d. 3 chords of the sphere
   e. a secant of the sphere
   f. a tangent of the sphere
   g. Write a definition of a diameter of a sphere.

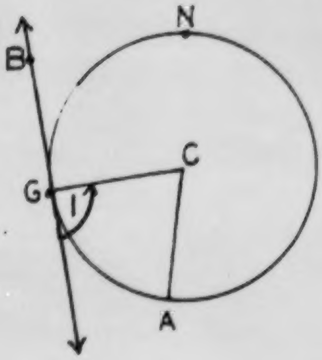


4. Refer to the diagram of the circle with center C. Fill in each blank to make a true statement.

   a. If $m\angle 1 = 90$, then $\overleftrightarrow{BG}$ is __?__ to the circle with center C.

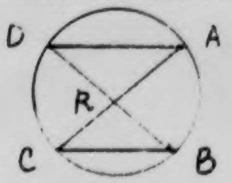
   b. If $\widehat{GA}^\circ = 80$, then $m\angle GCA =$ __?__.

   c. If $\widehat{GA}^\circ = 80$, then $\widehat{GNA}^\circ =$ __?__ .



(Go on to the next page.)

5. Refer to the diagram. Fill each blank with a true statement.

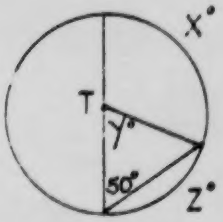


a. If $\widehat{AB}^\circ = 96$, then $m\angle ADB$ = __?__.

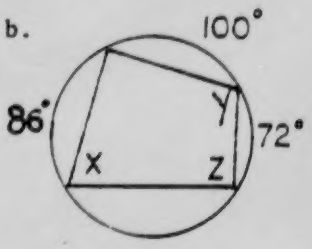
b. If $m\angle DAC = 48$, then $\widehat{DC}^\circ$ = __?__
and $m\angle B$ = __?__.

6. In each diagram, determine the values of x, y, and z.

a.



b.



7. Use $\pi = 3.14$ or $\frac{22}{7}$ to find the circumference of a circle whose diameter is:

a. 35 cm

b. 6 cm

8. Find the circumference of a circle whose radius is 5.2 cm.

9. Write a formula for the length of an arc ($m\ \widehat{AB}$) in terms of:

a. $\widehat{AB}^\circ$ and circumference C

b. $\widehat{AB}^\circ$ and radius r

c. $\widehat{AB}^\circ$ and diameter d

10. Find the lengths of the arcs if:

a. the circle has circumference 18 cm and the arc has degree measure 160.

b. the circle has diameter 2 dm and the arc has degree measure 120.

c. the circle has radius 3 cm and the arc has degree measure 40.

Unit VI Test Name _______________________________

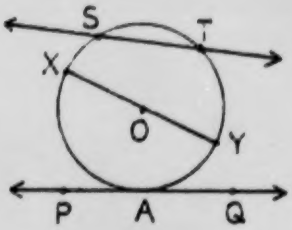
Place your answers in the space provided.

1. Define and draw a picture of each of the following:

   a. a tangent of a circle

   b. an inscribed angle of a circle

2. Refer to the diagram of the circle with center O. Name each of the following:



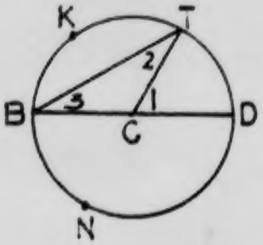
_______ a. a secant

_______ b. a diameter

_______ c. a radius

_______ d. a tangent

3. In the figure, $\overline{BD}$ is a diameter of the circle with center C and $\widehat{TD}^\circ = 40$. Find:



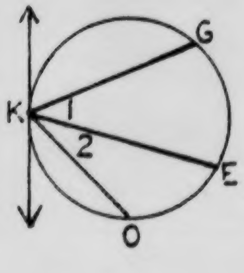
_______ a. $m\angle 1$

_______ b. $m\angle 3$

_______ c. $\widehat{BKT}^\circ$

_______ c. $\widehat{BNT}^\circ$

4. Refer to the diagram. Fill in each blank to make a true statement.



_______ a. If $\widehat{GE}^\circ = 80$, then $m\angle 1 =$ ___?___.

_______ b. If $m\angle 2 = 28$, then $\widehat{EO}^\circ =$ ___?___.

_______ c. If $m\angle 1 = 42$ and $m\angle 2 = 36$, then $\widehat{GEO}^\circ =$ ___?___ and $\widehat{GKO}^\circ =$ ___?___.

____

5. Use $\pi = 3.14$ or $\frac{22}{7}$ to find the circumference of a circle whose diameter is

_______ a. 4 meters

_______ b. 14 centimeters

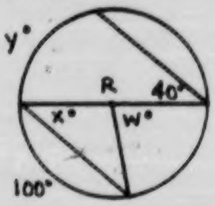
(Go on to the next page.)

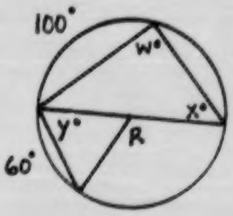
6. Find the length of an arc if the circle has

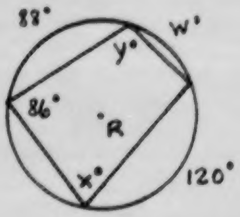
________ a. circumference 24 cm and the arc has degree measure 120.

________ b. diameter 4 dm and the arc has degree measure 90.

7. Determine the values of w, x, and y below for each circle with center R.







a. ________ b. ________ c. ________

________ ________ ________

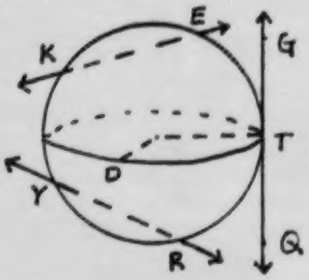
________ ________ ________

8. Refer to the diagram of a sphere with center O. Name each of the following:

________ a. a chord

________ b. a secant

________ c. a tangent

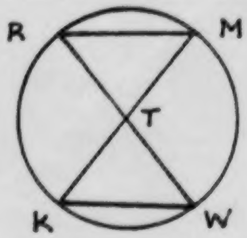


________ 9. Find the circumference of a circle whose radius is 6 cm.

10. Refer to the diagram of the circle with center T such that $\overline{RM} \parallel \overline{KW}$, $\widehat{RK}° = 108$. Find:

________ a. m ∠ K

________ b. m ∠ KTW



ANSWER KEY

Supplementary Worksheet 1

1. a. 3π cm b. 24π m c. 10π dm d. 40π cm

2. a. about 88 cm b. about 188.4 cm

3. about 94.20 m

4. a. 50 b. 180 c. 130 d. 310

5. a. $\frac{8\pi}{9}$ m b. 2π mm c. 5 cm d. 11π cm

(1 point each)

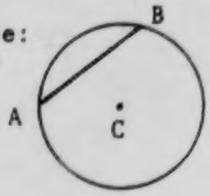
Quiz 1 *(10 points)*

1. a diameter 2. a chord 3. a radius

4. a tangent 5. a secant

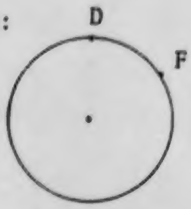
6. $\overset{\frown}{AB}$ or $\overset{\frown}{BC}$ 7. $\overset{\frown}{BAC}$ or $\overset{\frown}{BCA}$ 8. $\overset{\frown}{ABC}$ or $\overset{\frown}{AC}$

9. Answers may vary. Here is an example: A chord is a line segment that joins two points on a circle.



$\overline{AB}$ is a chord.

10. Answers may vary. Here is an example: A minor arc is an arc less than a semicircle.



$\overset{\frown}{DF}$ is a minor arc.

(1 point each)

Quiz 2 *(10 points)*

1. a. 87.92 cm or 88 cm b. 15.7 cm

2. 25.12 cm

3. a. 90 b. 145 c. 125

4. 3 cm

5. a. 50 b. 40 c. 80

(Go on to the next page.)

Take-home Test Answers will vary.

Review Worksheet

1. Answers will vary.

2. a. Any three out of: $\overline{CN}$, $\overline{CF}$, $\overline{CR}$, $\overline{CG}$
   b. $\overline{GF}$ or $\overline{NR}$
   c. $\overleftrightarrow{NF}$, $\overleftrightarrow{NR}$, or $\overleftrightarrow{FG}$
   d. $\overleftrightarrow{LD}$ or $\overleftrightarrow{JW}$
   e. $\overline{NF}$
   f. Point F or G

3. a. Any four out of: E, N, V, J, U, Y, A, P
   b. $\overline{AO}$, $\overline{NO}$, $\overline{OP}$
   c. $\overline{AP}$
   d. $\overline{AP}$, $\overline{JV}$, $\overline{EU}$
   e. $\overleftrightarrow{JV}$
   f. $\overleftrightarrow{KS}$
   g. A diameter of a sphere is a chord that passes through the center of the sphere.

4. a. tangent b. $m \angle GCA = 80$ c. $\widehat{GNA}^\circ = 280$

5. a. $m \angle ADB = 48$ b. $\widehat{DC}^\circ = 96$, $m \angle B = 48$

6. a. $x = 100$, $y = 80$, $z = 80$
   b. $x = 86$, $y = 94$, $z = 93$

7. a. 110 cm b. 18.84 cm

8. 10.4π cm or 32.656 cm

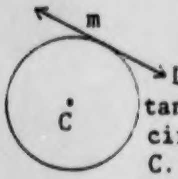
9. a. $m\widehat{AB} = C \cdot \frac{\widehat{AB}^\circ}{360^\circ}$ b. $m\widehat{AB} = 2\pi r \cdot \frac{\widehat{AB}^\circ}{360^\circ}$
   c. $m\widehat{AB} = \pi d \cdot \frac{\widehat{AB}^\circ}{360^\circ}$

10. a. 8 cm b. $\frac{2}{3}\pi$ dm c. $\frac{2}{3}\pi$ cm

Unit VI Test *(50 points)*

(4) 1. Answers may vary slightly. Here are possible answers:

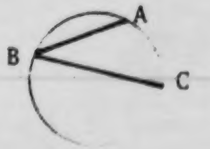
a. A tangent of a circle is a line or line segment in the plane of a circle, that intersects the circle in exactly one point.



Line m is a tangent of the circle with center C.

(Go on to the next page.)

b. An inscribed angle of a circle is an angle whose vertex lies on the circle and whose sides are chords of the circle.



$\angle$ ABC is an inscribed angle.

(4) 2. a. $\overleftrightarrow{ST}$ c. $\overline{OX}$ or $\overline{OY}$

b. $\overline{XY}$ d. $\overleftrightarrow{PQ}$ or $\overleftrightarrow{PA}$ or $\overleftrightarrow{AQ}$

(8) 3. a. $m\angle 1 = 40$ c. $\overset{\frown}{BKT}^\circ = 140$

b. $m\angle 3 = 20$ d. $\overset{\frown}{BNT}^\circ = 220$

(8) 4. a. $m\angle 1 = 40$

b. $\overset{\frown}{EO}^\circ = 56$

c. $\overset{\frown}{GEO}^\circ = 156$, $\overset{\frown}{GKO}^\circ = 204$

(4) 5. a. about 12.56 meters b. about 44 centimeters

(4) 6. a. 8 cm b. π dm, or 3.14 dm, or $\frac{22}{7}$ dm

(9) 7. a. $w = 80$ b. $w = 90$ c. $w = 52$

$x = 40$ $x = 50$ $x = 70$

$y = 80$ $y = 60$ $y = 110$

(3) 8. a. $\overline{EK}$ or $\overline{YR}$ b. $\overrightarrow{EK}$ or $\overleftarrow{YR}$ c. $\overleftrightarrow{GQ}$

(2) 9. 12π cm or 37.68 cm

(4) 10. a. $m\angle K = 54$ b. $m\angle KTW = 72$

Unit VII: Areas and Volumes

Objectives

(1) To discover and use the formula for the area of a rectangular region.

(2) To find the area of a polygonal region.

(3) To discover and use the formula for the area of a triangular region.

(4) To discover and use the formula for the area of a parallelogram.

(5) To discover and use the formula for the area of a circle.

(6) To investigate the relationship between the area of a rectangular region and its perimeter.

(7) To discover and use the formulas for the volumes of a rectangular prism, a triangular prism, a cylinder, and a right circular cone.

Materials Needed

Set 1:	metric ruler
Set 2:	metric ruler, square centimeter grid sheet
Set 3:	ruler, compass, scissors, tape
Set 4:	A piece of string about 38 cm long
Set 5:	boxes, cylindrical can, compass, stiff paper, scissors, tape, sand

Suggestions for Teaching

Set 1, pages 180 - 184

Vocabulary: plane and space regions, square centimeter, area, rectangular region, polygonal region, nonoverlapping

Materials Needed: metric ruler

Comments: Have students read pages 180 - 181 and do Exercises 1 - 5 on their own. Then discuss the reading and their answers to the exercises as a class. The following properties for the measures of plane regions should be emphasized in the discussion:

1) Two plane regions with the same area can have different shapes.
2) Any plane region (circle, triangle, rectangle, square, etc.) can be used as the unit of measure of plane regions.

3) The length of a rectangular region multiplied by its width is equal to the area of the region.

Instruct students to take measurements of the regions in Exercises 6 and 12 to the nearest tenth of a centimeter.

SQUARE ROOTS REVIEW

Students may need a review of square roots for some of the work in this set and in Set 3.

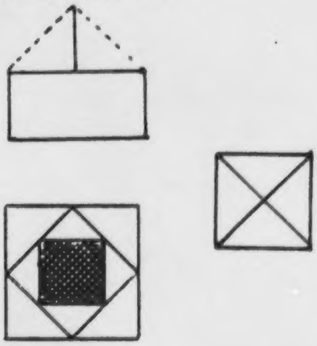
Have students discuss their answers to Exercises 8 - 11. Be sure to emphasize the properties of area covered in these exercises.

MORE THAN ONE WAY

Be sure to have students explain how they obtained their answers for Exercise 12. They should realize that there may be more than one way to find the area of regions like those in Exercises 12a - c.

You may want to have students do the following activity:

Take a square sheet of paper. Fold the corners into the center to form another square as shown. Fold the corners in again to form a smaller square. Now unfold the sheet. How does the area of the smallest square compare with the area of the original square? Suppose the side of the original square were 10 cm. What would be the area of the smallest square?



Set 2, pages 185 - 188

Materials Needed: metric ruler, square centimeter grid sheet

Comments: You may want to have the students do Exercises 1 - 16 as a class. Have a student demonstrate on the overhead projector as the others follow along. (Transparency masters of the figures for these exercises are provided.) Then have the students do Exercises 17 and 18 and the supplementary worksheet on their own.

Provide each student with a square centimeter grid sheet to do Exercise 1. They can use the grid to help them count the number of square centimeters required to cover the triangular regions.

MEASURE TO THE NEAREST...?

Instruct students to measure the figures in exercise 1 to the nearest centimeter. Measurements for Δ ABC on page 186 should be taken to the nearest tenth of a centimeter.

A supplementary worksheet which contains exercises like those in Sets 1 and 2 is provided.

QUIZ I   Anytime after Set 2

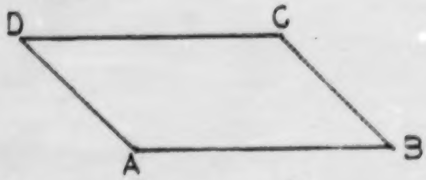
Set 3, pages 189 - 192

Materials Needed: ruler, compass, protractor, scissors, tape

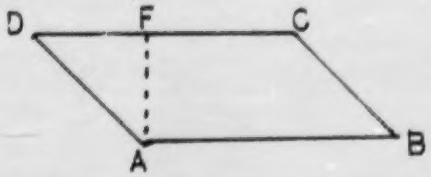
Comments: Have the students follow steps 1 - 7 on page 189 to discover the formula for the area of a parallelogram. Have them discuss their results before assigning Exercises 8 and 9.

You may want to show the class an alternative method for finding the area of a parallelogram:

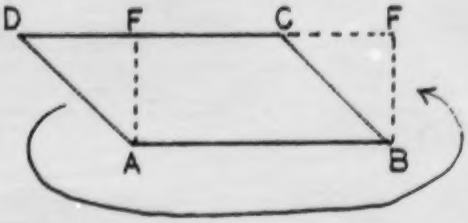
Suppose you are given parallelogram ABCD.



1) Draw an altitude from point A to $\overline{DC}$ as shown. Call the altitude $\overline{AF}$.



2) Cut out triangle ADF and place it on the other side as shown. Notice that the rectangle formed has the same area as parallelogram ABCD.

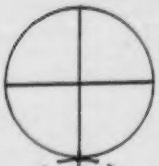


3) The area of the rectangle is equal to Ab x AF. Thus, the area of the parallelogram is AB x AF. In general, if b is the base of the parallelogram and a is an altitude to the base, then the area of the parallelogram is ab.

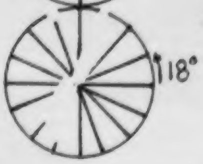
Be sure to have students explain how they obtained their answers to Exercises 8 and 9. (Transparency masters for these exercises are provided.)

You may want to have the students do the circle activity on pages 191 - 192 as a class. One way to draw the 10 diameters equally spaced around the circle is described on the next page.

1) Fold the circle into quarters as shown.



2) Using the protractor measure 18° angles as shown. Then draw the diagonals.

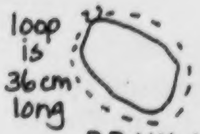


An alternative may be to have the students estimate where the diameters may be.

Set 4, pages 193 - 194

Materials Needed: a piece of string about 38 cm long

Comments: You may want to have students work in pairs to answer the exercises in this set.



For Exercise 1, students need a piece of string that is a little more than 36 centimeters long. Some "slack" is needed to tie the ends of the string together to form a loop 36 cm long.

DRAW A PICTURE

Remind students to draw pictures of the rectangle if they need help in completing the table in Exercise 4!

STUDY TABLES

Students are to study the tables they completed in Exercises 1 - 4 to answer Exercises 5 - 10.

QUIZ 2

Anytime after Set 4

Set 5, pages 195 - 201

Vocabulary: volume, cubic unit, right circular cylinder, right circular cone, slant height, cross-section

Comments: First have the students read page 195 and answer Exercises 1 - 8 on their own. Then discuss their answers as a class.

Students should be cautioned about problems like Exercise 8c. The measurements must be in the same unit. For Exercise 8c, either the area of the base must be converted to 2000 mm^2 or the height of the prism must be converted to 3.5 cm.

"REAL" EXAMPLES

To review the meaning of volume and the formula for finding the volume of a prism, show the class some boxes and ask them to estimate their volume. Then have them find the actual volume.

Compare the different units of measure. Point out the significance of the words, "square" and "cubic."

Exercises 9 and 10 should be discussed as a class before assigning Exercises 11 thourgh 13. Exercise 13 is a good one for problem solving. Be sure to have the students discuss how they obtained their answers.

You may want to have students work in pairs to make the model of a cone, (described on page 200), and to answer Exercises 17 through 20.

Take-home Problem

Assign this problem for homework.

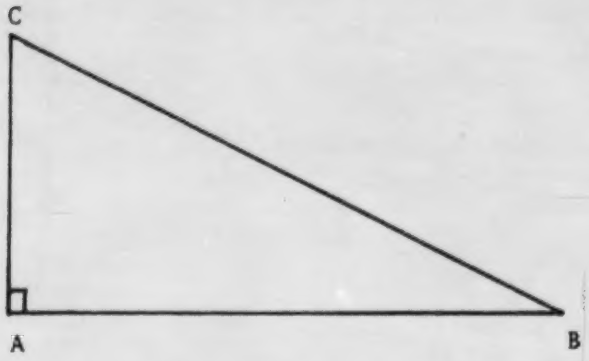
Review

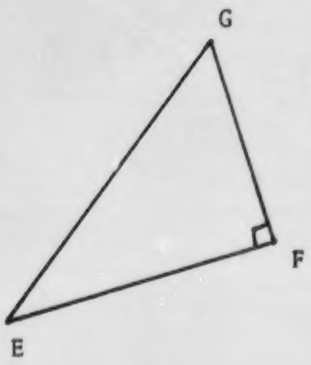
A review worksheet for this unit is provided.

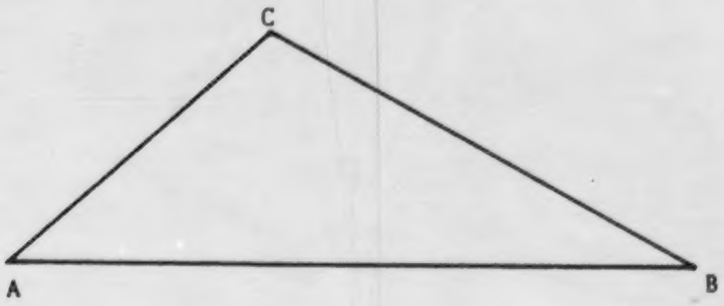
UNIT VII TEST

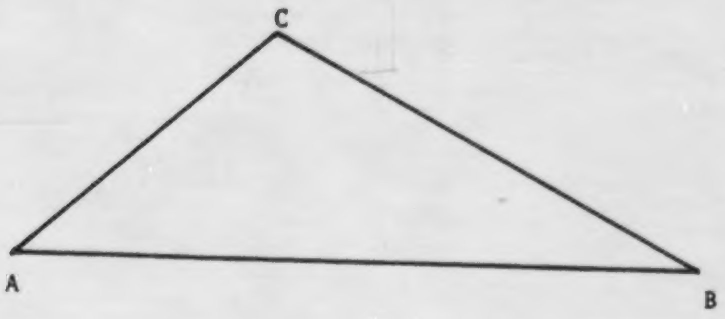
SQUARE CENTIMETER GRID SHEET

Transparency 1
Unit VII. Set 2





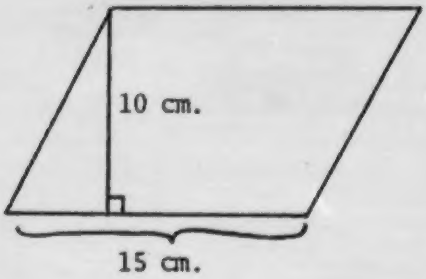




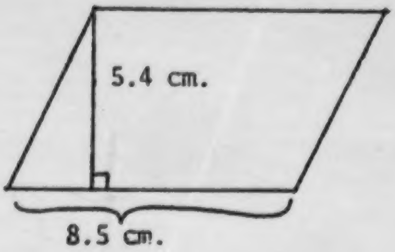
Transparency 2
Unit VII   Set 3

8. Use the formula you found in Exercise 7 to find the areas of the parallelograms.

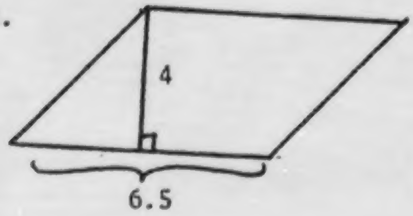
a.



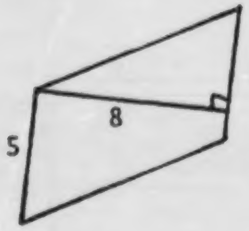
b.



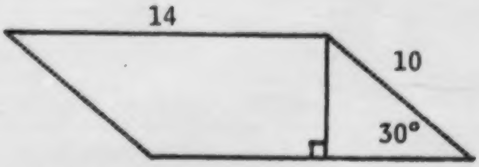
c.



d.

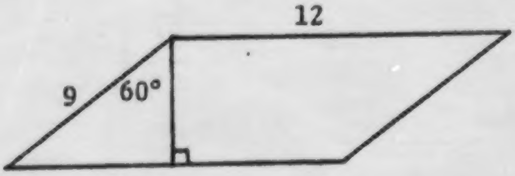


e.

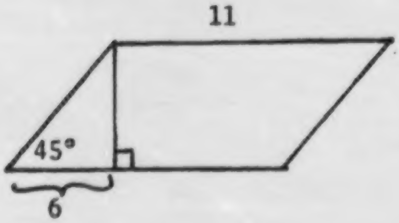


(Recall a special right triangle.)

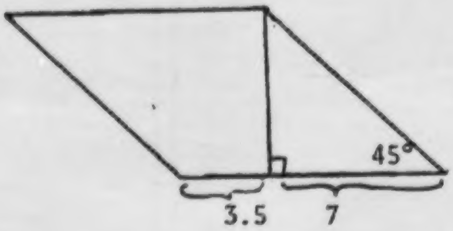
f.



g.

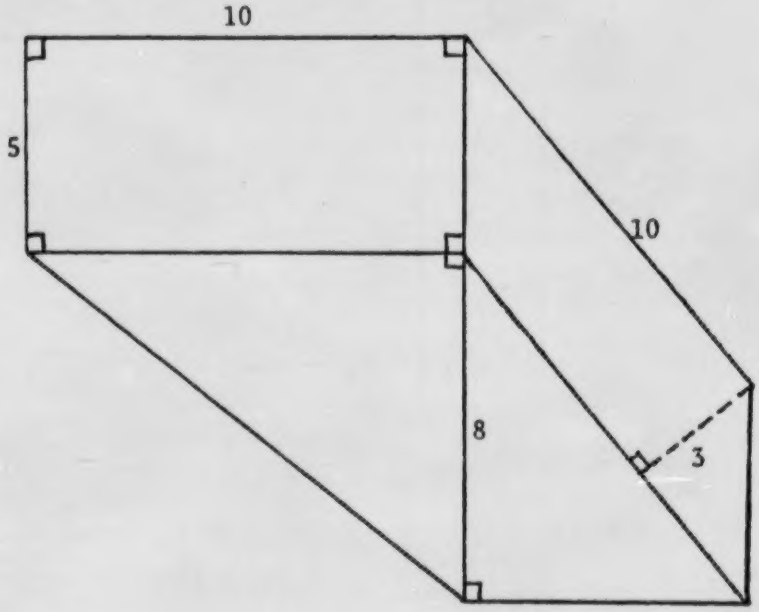


h.



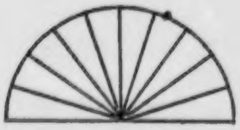
From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 190.

9. Assume that the figure below can be dissected into a rectangle, a parallelogram, and two right triangles as shown. Find the area of the entire region.

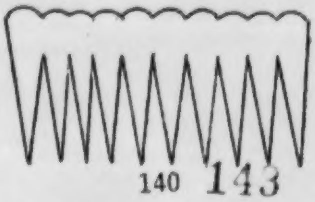


The shape of a circular region is quite different from a parallelogram. However, you can use your knowledge of the area of a parallelogram to arrive at the area of a circle.

On a separate sheet of paper, draw a circle with diameter 10 centimeters long. Draw 10 diameters which are equally spaced around the circle. Cut the region in half along one diameter and cut slits along the radii as shown below.



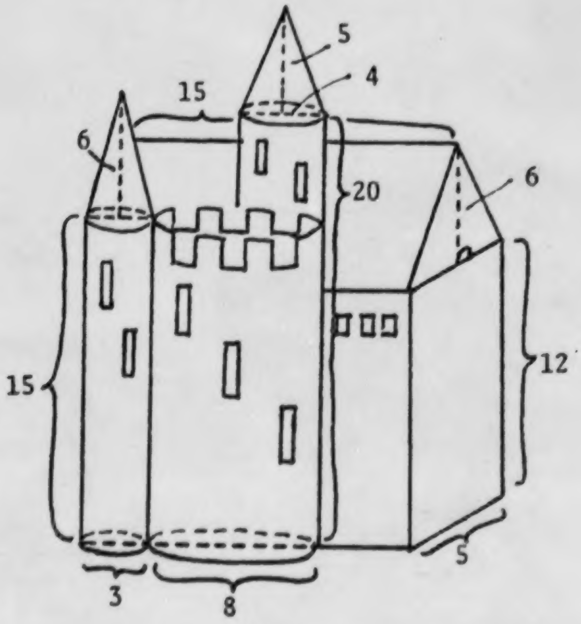
Open the semi-circular region out and tape it down on another sheet of paper as shown below.



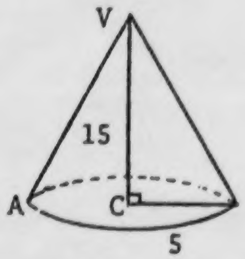
From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 191.

Transparency 4
Unit VII Set 5

13. Here is a model of part of a castle. Assume it is solid, ignore the openings, and find its volume.



Shown below is a right circular cone.



It has a circular region as base. V is the vertex of the cone, C is the center of the circle, and A is a point on the circle. In a right circular cone, $\overline{VC}$ is perpendicular to the plane of the circle. The length of this perpendicular segment is the height of the cone. The distance between the vertex and a point on the circle (for example VA) is called the slant height. In the pictured cone, the radius of the circular base is 5 cm and the height is 15 cm.

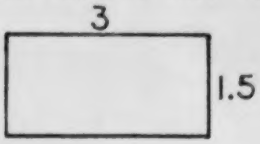
From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 199.

Unit VII, Sets 1 and 2 Name________________________

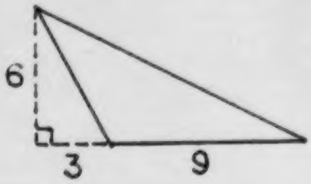
Supplementary Worksheet 1

Find the area of each region below, using the measures indicated.

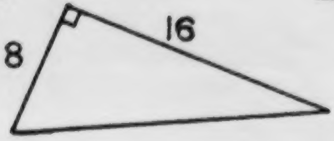
________ 1.



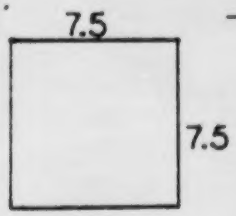
________ 2.



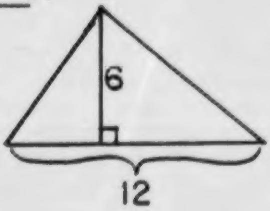
________ 3.



________ 4.



________ 5.



Find the area for each of the following regions described below:

6. A triangle with

________ a. base = 10 cm and altitude = 5.5 cm

________ b. base = 25 cm and altitude = 8 cm

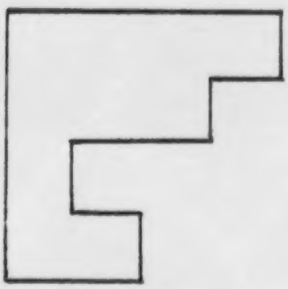
7. A rectangle with

________ a. length = 6 cm and width = 3.5 cm

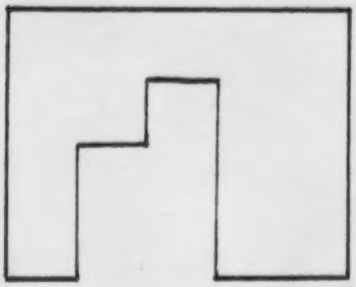
________ b. length = 14½ m and length = 8¼ m

Find the area of the large regions. Take measurements to the nearest centimeter.

________ 8.



________ 9.

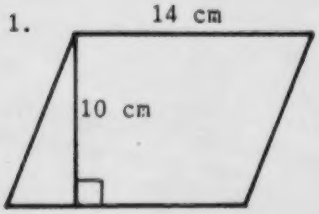


Unit VII, Sets 1 - 3
Supplementary Worksheet 2

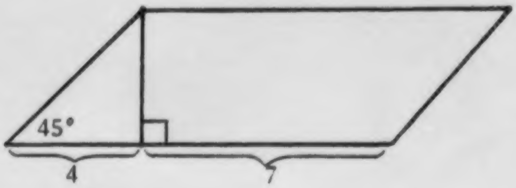
Name ______________________________

Find the area of the regions below, using the measures indicated:

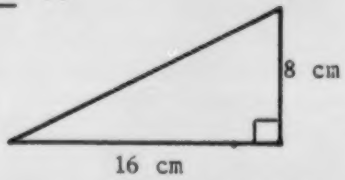
______ 1.



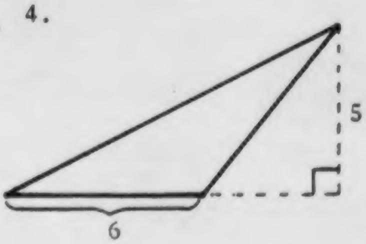
______ 2.



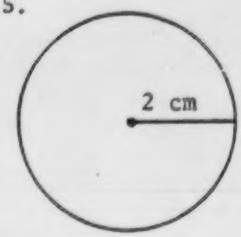
______ 3.



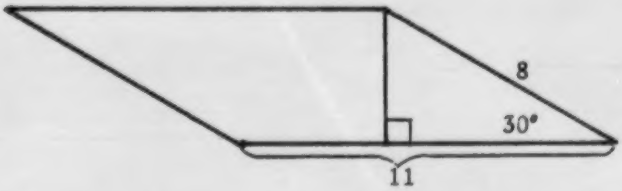
______ 4.



______ 5.



______ 6.



7. Find the area of the circle if its

______ a. radius is 10 cm

______ b. diameter is 18 cm

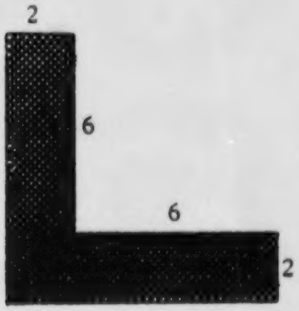
______ 8. Find the area of a parallelogram whose base is 25 cm and height is 8 cm.

______ 9. If the circumference of a circle is 8π, what is its area?

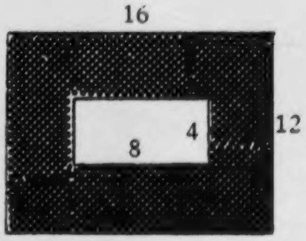
(Go on to the next page.)

Find the area of each shaded region.

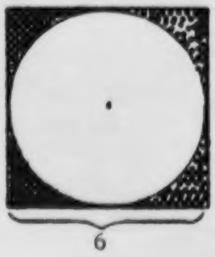
________10.



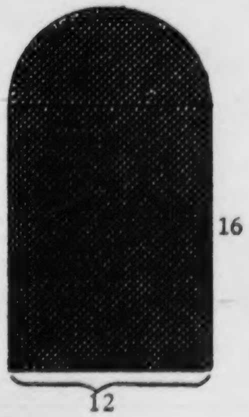
________11.



________12.



________13.



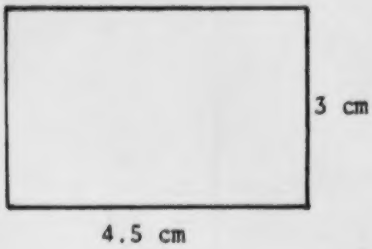
Unit VII

Name ______________________________

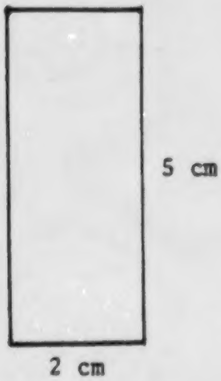
Quiz 1

1. Find the area of each region below, using the measures indicated.

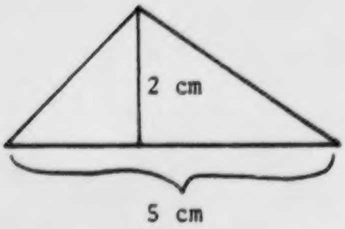
________ a.



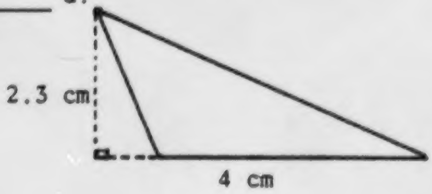
________ b.



________ c.



________ d.



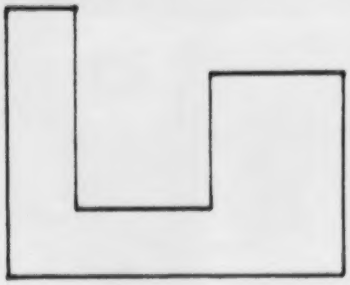
2. Find the areas of the following:

________ a. a rectangle with length 9.5 cm and width 4 cm

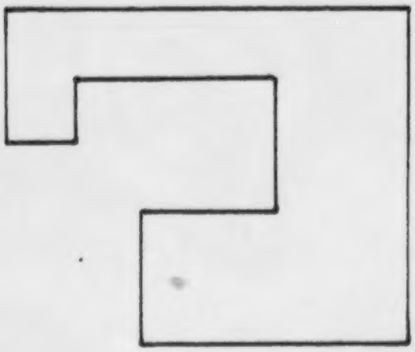
________ b. a triangle which has a height of 8 cm and base length of 25 cm

3. Find the areas of the large regions. Take measurements to the nearest centimeter.

________ a.



________ b.

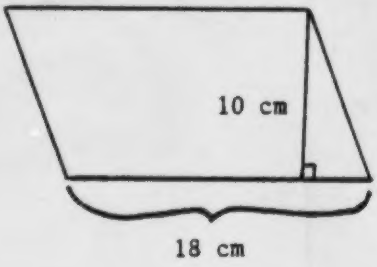


Unit VII Name______________________________

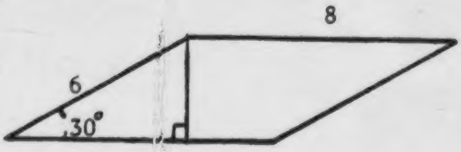
Quiz 2

1. Find the area of each parallelogram below.

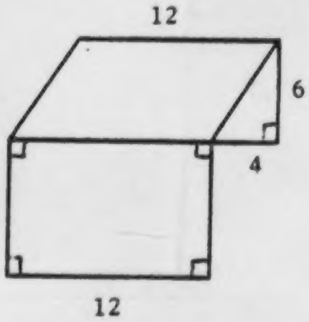
________ a.



________ b.



________ 2. Find the area of the entire region.



3. Find the area of the circle if its

________ a. radius is 6 cm

________ b. diameter is 8 cm

4. Find the perimeters and areas of the following rectangles:

	Length	Width	Perimeter	Area
a.	8	5	________	________
b.	15	7	________	________

Unit VII Name________________________________

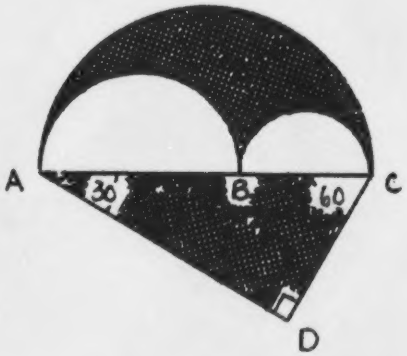
Take-home Problem

Shown below are three semicircles and a 30-60 right triangle. The diameters of the smaller circles are:

AB = 12

BC = 8

Find the area of the shaded portion.

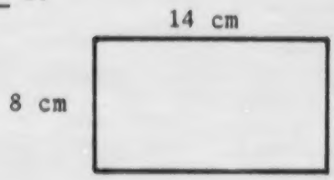


Unit VII Name________________________________

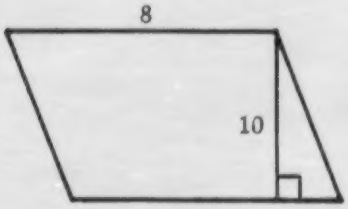
Review Worksheet

1. Find the area of each region below, using the measures indicated.

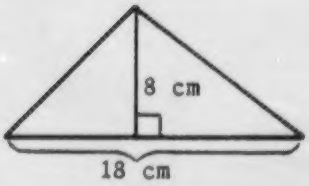
________ a.



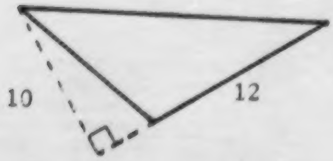
________ b.



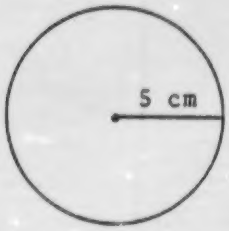
________ c.



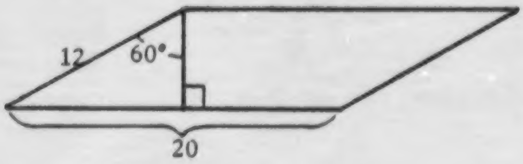
________ d.



________ e.



________ f.



2. Find the area of the regions described below:

________ a. a triangle with base length = 10 cm and height = 6 cm

________ b. a circle with radius 7 cm

________ c. a rectangle with length = 15 m and width = 8 m

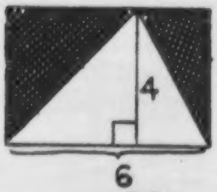
________ d. a parallelogram with base length = 24 dm and height = 7 dm

________ e. a circle with diameter 24 mm

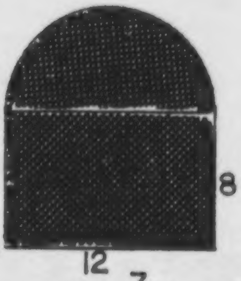
(Go on to the next page.)

3. Find the area of the shaded regions.

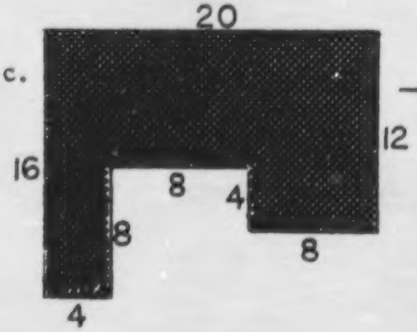
________ a.



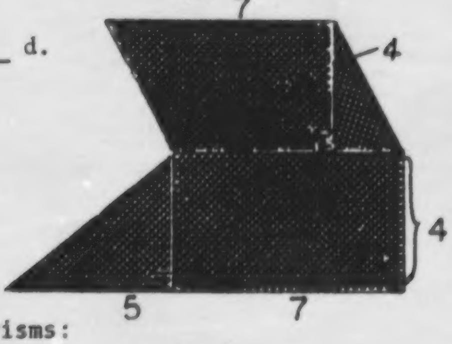
________ b.



________ c.



________ d.



4. Find the volume of the following prisms:

________ a. a rectangular prism with length = 8 m, width = 6 m, and height = 4 m

________ b. a triangular prism with base of triangle = 6 cm, height of triangle = 3 cm, and height of prism = 8 cm

________ c. a rectangular prism with length = 5 cm, width = 3 cm, and height = 4 cm

________ d. a triangular prism with base of triangle = 10 cm, height of triangle = 5 cm, and height of prism = 9 cm

5. Find the volume of a cylinder if:

________ a. the radius of its base is 5 cm and its height is 12 cm

________ b. the radius of its base is 2 dm and its height is 8 dm

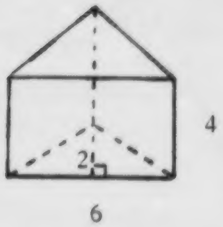
6. Find the volume of a right circular cone if its:

________ a. radius is 9 cm and height is 20 cm

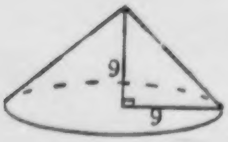
________ b. radius is 4 cm and height is 8 cm

7. Find the volume of each figure:

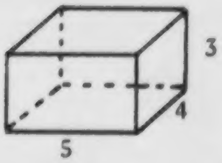
_______ a.



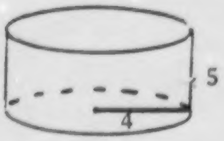
_______ b.



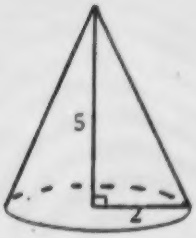
_______ c.



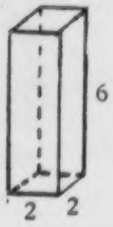
_______ d.



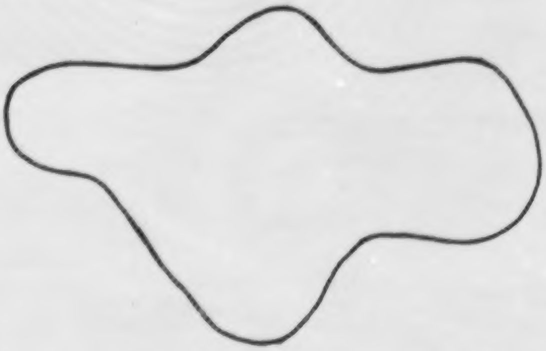
_______ e.



_______ f.



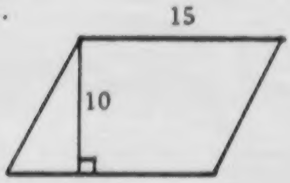
_____ 8. Estimate the area of the following region. (Be sure to indicate your unit of measure.)



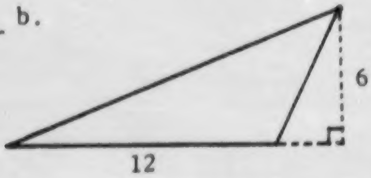
Unit VII Test Name ______________________________

1. Find the area of each region below, using the measures indicated.

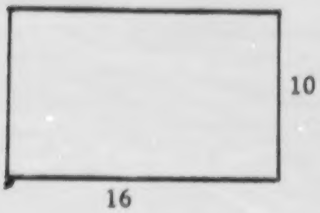
________ a.



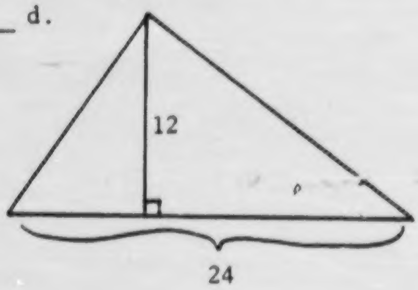
________ b.



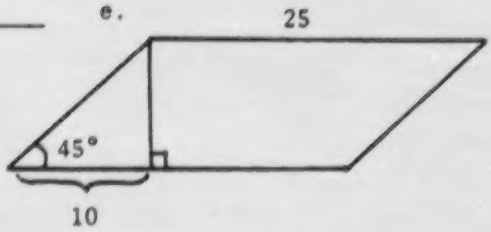
________ c.



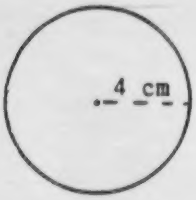
________ d.



________ e.



________ f.



2. Find the area of the regions described below:

________ a. a triangle with base length = 24 cm and height = 15 cm

________ b. a rectangle with length = 16 cm and width = 7 cm

________ c. a parallelogram with base length = 20 cm and height = 9 cm

3. Find the area of a circle (in terms of π) if its

________ a. diameter is 10 cm

________ b. radius is 9 cm

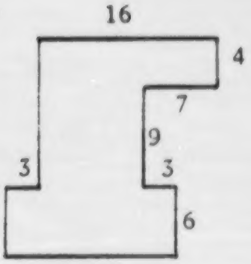
________ 4. Find the perimeter and area of a rectangular region whose length is 12 cm and width is 5 cm.

____

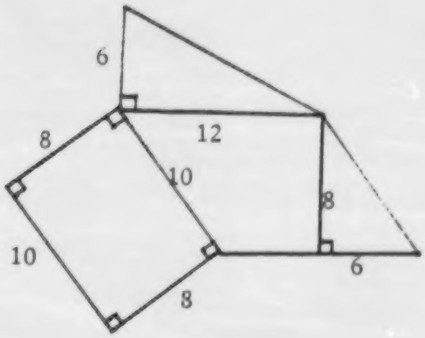
(Go on to the next page.)

5. Find the area of the entire region:

________ a.

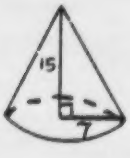


________ b.

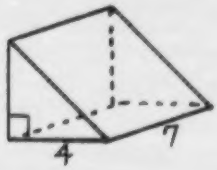

6
12
8
10
8
10
8
6

6. Find the volume of each figure, using the measures indicated.

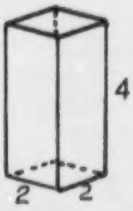
________ a.



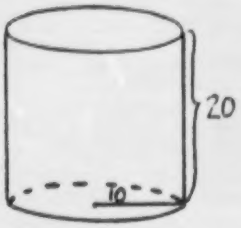
________ b.



________ c.



________ d.



7. Find the volume of the following prisms:

________ a. a rectangular prism with
length = 8m , width = 5 m, and height = 9 m

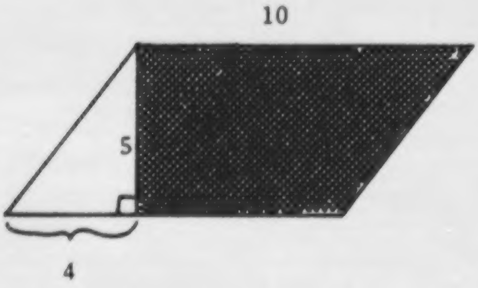
________ b. a triangular prism with
base of triangle = 8 cm, height of triangle = 6 cm, and
height of prism = 7 cm

(Go on to the next page.)

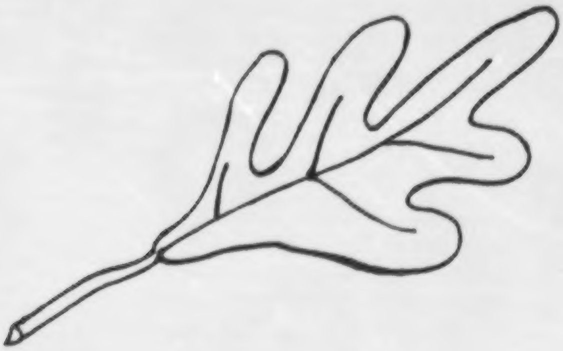
________ 8. Find the volume of a cone if the radius of its base is 3 cm and the height of the cone is 10 cm.

________ 9. Find the volume of a cylinder if the radius of its base is 3 cm and its height is 12 cm.

________10. Find the area of the shaded region below:



________11. Estimate the area of the following leaf.
(Be sure to indicate your unit of measure.)



ANSWER KEY

Supplementary Worksheet 1

1. 4.5 2. 27 3. 64 4. 56.25 5. 36
6. a. 27.5 sq. cm b. 100 sq. cm
7. a. 21 sq. cm b. 119.625 sq. m
8. 10 sq. cm 9. 15 sq. cm

Supplementary Worksheet 2

1. 140 sq. cm 2. 28 3. 64 sq. cm
4. 15 5. 4π sq. cm 6. 44
7. a. 100π sq. cm b. 81π sq. cm
8. 200 sq. cm 9. 16π
10. 28 11. 160 12. $36 - 9\pi$ 13. $18\pi + 192$

Quiz 1 *(10 points)*

(4) 1. a. 13.5 sq. cm b. 10 sq. cm
c. 5 sq. cm d. 4.6 sq. cm
(2) 2. a. 38 sq. cm b. 100 sq. cm
(4) 3. a. 12 b. 19

Quiz 2 *(10 points)*

(2) 1. a. 180 sq. cm b. 24
(2) 2. 180
(2) 3. a. 36π b. 16π
(4) 4. a. Perimeter = 26, Area = 40
b. Perimeter = 44, Area = 105

Take-home Problem $24\pi + 50\sqrt{3}$

Review Worksheet

1. a. 112 sq. cm b. 80 c. 72 sq. cm
d. 60 e. 25π sq. cm f. 120
2. a. 30 sq. cm b. 49π sq. cm c. 120 sq. m
d. 168 sq. dm e. 144π sq. mm
3. a. 12 b. $18\pi + 96$ c. 224 d. 66
4. a. 192 cubic m b. 72 cubic cm
c. 60 cubic cm d. 225 cubic cm
5. a. 300π cubic cm b. 32π cubic dm
6. a. 540π cubic cm b. $\frac{128}{3}\pi$ cubic cm
7. a. 24 b. 243π c. 60
d. 80π e. $\frac{20}{3}\pi$ f. 24

8. Answers will vary.

(Go on to the next page.)

Unit VII Test *(50 points)*

(12) 1. a. 150 b. 36 c. 160
d. 144 e. 250 f. 16π sq. cm
(6) 2. a. 180 sq. cm b. 112 sq. cm c. 180 sq. cm
(4) 3. a. 25π sq. cm b. 81π sq. cm
(2) 4. Perimeter = 34 cm , Area = 60 sq. cm
(6) 5. a. 235 b. 212
(8) 6. a. 245π b. 56 c. 16 d. 2000π
(4) 7. a. 360 cubic m b. 168 cubic m
(4) 8. 30π cubic cm 9. 108π cubic cm
(2) 10. 40
(2) 11. Answers will vary.

Unit VIII: Transformations

Objectives

(1) To identify reflection congruent figures.

(2) To find the image of a figure under a given reflection.

(3) To find the axis of reflection for a pair of reflection congruent figures.

(4) To identify translation congruent figures.

(5) To find the image of a figure under a given translation.

(6) To find the arrow which represents a translation given a pair of translation congruent figures.

(7) To identify rotation congruent figures.

(8) To find the image of a figure under a given rotation.

(9) To find the center of rotation and the directed arc given rotation congruent figures.

(10) To identify glide reflection congruent figures.

(11) To find the image of a figure under a given glide reflection.

(12) To explore the relationship among the four transformations: reflection, translation, rotation, and glide reflection.

Materials Needed

Tracing paper and a straightedge will be needed for all the sets. A compass will also be needed for Set 3.

Suggestions for Teaching

Set 1, pages 204 - 208

Vocabulary:	transformation, reflection, reflection congruent, axis, orientation, opposite orientation, image, fixed point, fixed line
Materials Needed:	tracing paper, straightedge

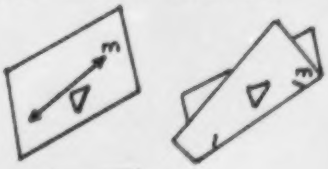
Geometric transformations and the manipulation of tracing paper are both probably new to the students. However, demonstrations by the teacher are not always recommended. For this set, it is suggested that you work with the students, as much as possible, helping them to read, reread, and follow directions rather than show them what to do and have them duplicate it.

Be sure to have class discussions about the reading and assignment. Try to elicit from the students the properties of reflection congruent figures. Have them explain how to find the images of figures under given reflections as well as how to find the axis of reflection for reflection congruent figures. Students can easily demonstrate these on the overhead. Every step of the tracing method can be shown on the overhead except "trace over the figure to get an impression."

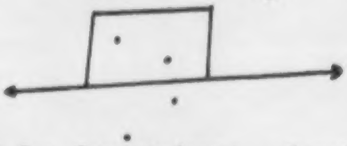
Encourage students to use the vocabulary introduced in this set.

You may want to ask students if they know other methods (besides the tracing method) for finding the image of a figure under a reflection. Some students may be familiar with the following methods:

1) paper folding method:



2) mirror method:



Both methods are easy and fun for students. Supplementary Worksheets 1 and 2 may be assigned to provide practice in using these methods or to provide more practice in using the tracing method.

Set 2, pages 209 - 212

Vocabulary: translation, slide, translation congruent

Materials Needed: tracing paper, straightedge

Comments: Demonstrations by the teacher are not recommended again. As much as possible, you should work with the students, helping them to read, reread, and follow directions. Encourage peer teaching too.

Be sure to have class discussions, where students are given the opportunity to:

1) use the new vocabulary.
2) explain how to find the image of a figure under a given translation.
3) explain how to recognize translation congruent figures and how to find the arrow to represent the translation.
4) discuss the properties of translation congruent figures and of reflection congruent figures.

A worksheet that contains more exercises on finding images of figures under given translations is provided.

QUIZ I Anytime after Set 2

Set 3, pages 213 - 217

Vocabulary: rotation, rotation congruent figures

Materials Needed: tracing paper, straightedge, compass

Comments: A demonstration with an overhead may be helpful for page 213. For example, make two transparencies: 1) of all figures on page 213 and 2) of point 0, Δ ABC, and ▱ EFGH. Make a pinpoint hole at 0 in transparency 2. Place transparency 2 on top of transparency 1, matching 0, Δ ABC, and ▱ EFGH. Place a compass point on 0, through the hole. Then rotate the top transparency until Δ ABC fits on Δ A'B'C'. The same rotation makes ▱ EFGH fit on ▱ E'F'G'H'.

A demonstration may also be helpful for page 216. For example, make two transparencies of: 1) all figures on page 216 and 2) all figures except the directed arc. Make a pinpoint hole at 0 on the second transparency. Place Transparency 4 on top of Transparency 3, matching 0 and the figures. Place a compass point on 0, through the hole. Mark a dot at the tail end of the directed arc. Then rotate the top transparency until the dot is on the head of the arc. The figures on the top transparency are the images of the given figures on the bottom transparency.

As in the previous sets, class discussions are important. Be sure to have the students discuss the properties of rotation congruent figures and compare them with the properties of reflection congruent and translation congruent figures.

Ask students to demonstrate how to find the images of figures under given rotations and to find the center of rotation and the directed arc for rotation congruent figures.

Supplementary worksheets are provided for students who need more practice in finding the images of figures under given rotations and/or in identifying rotation congruent figures.

Set 4, pages 218 - 219

Vocabulary: glide reflection, slide reflection, glide reflection congruent figures

Materials Needed: tracing paper, straightedge

Comments: As in Sets 1 and 2, it is recommended that you try to work with the students on the exercises, rather than demonstrate what to do and then have them duplicate it. Class discussion are important. Have students discuss the properties of glide reflection congruent figures and compare them with the properties of reflection congruent, translation congruent, and rotation congruent figures.

Ask students to demonstrate how to find the images of figures under given glide reflections and to identify glide reflection congruent figures.

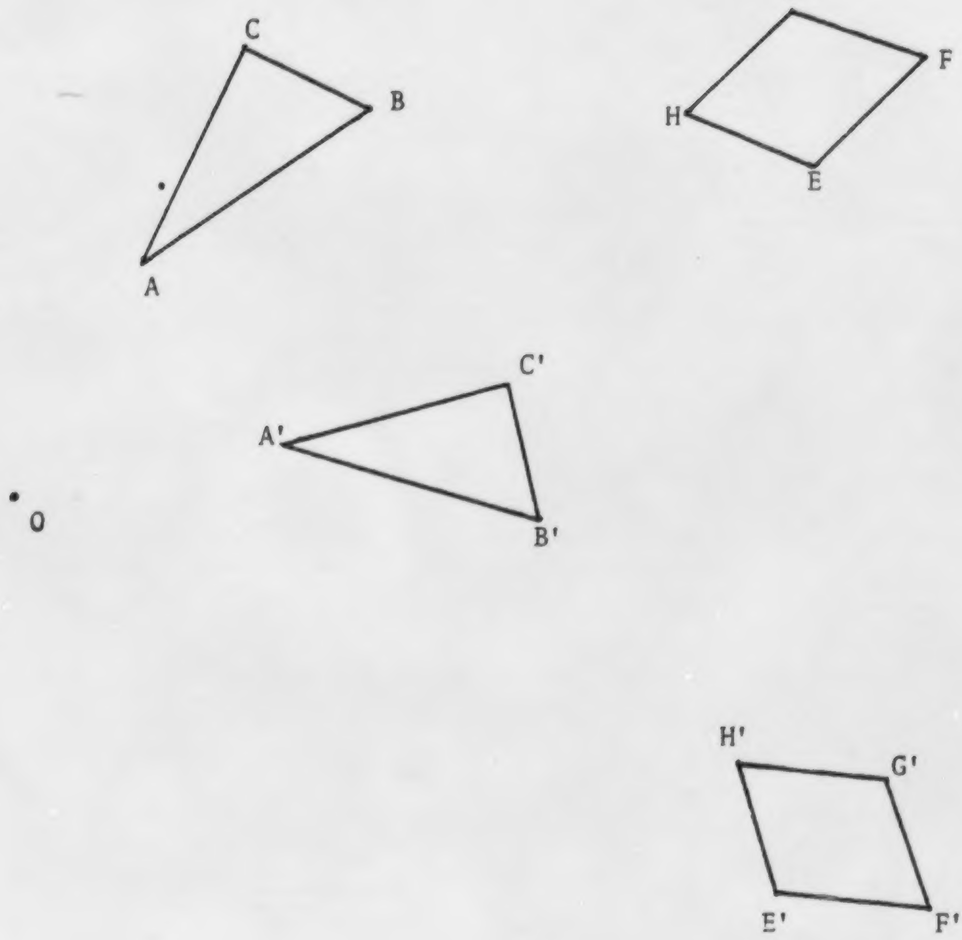
Set 5, page 220

Comments: You may want to have the students work in pairs to do Exercises 1 and 2. Have them come together as a class to discuss their results. Exercise 3 is optional and is intended as exploration for the more capable student.

Review Worksheet

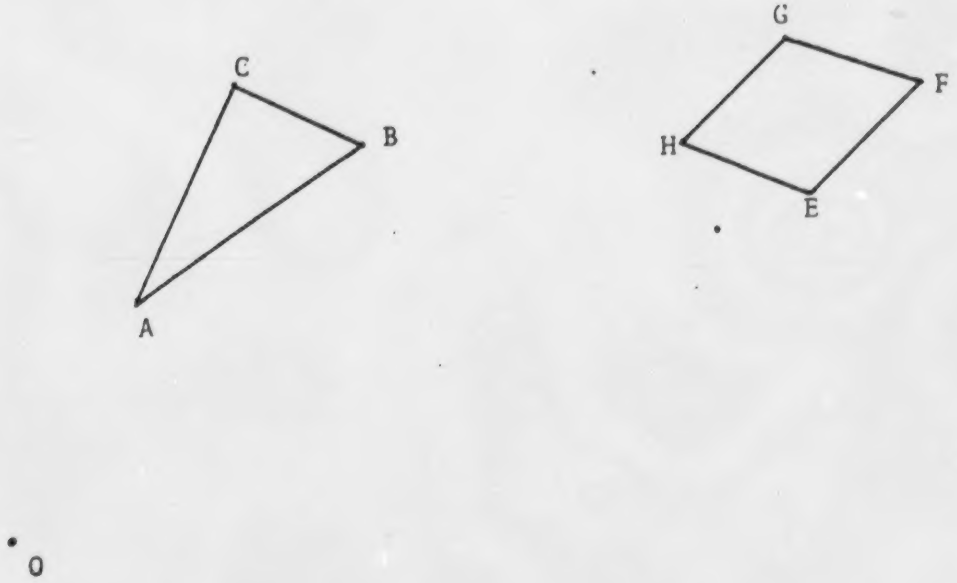
UNIT VIII TEST

Transparency 1
Unit VIII, Set 3

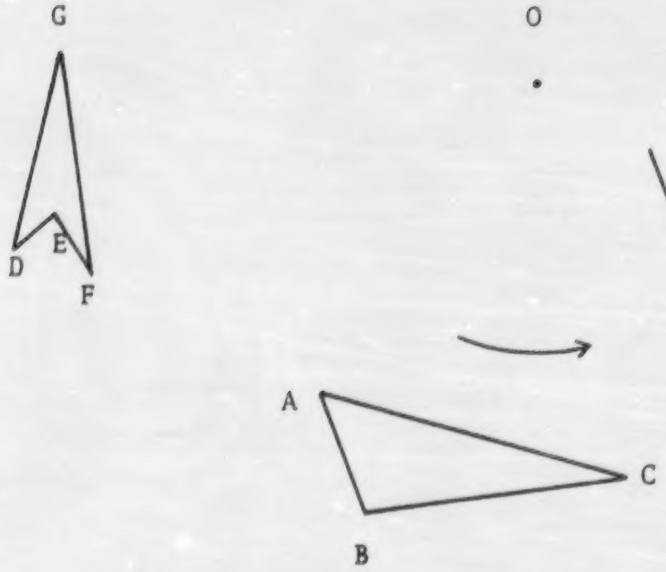

G
C
F
B
H
E
A
C'
A'
O
B'
H'
G'
E'
F'

From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 213.

Transparency 2
Unit VIII, Set 3

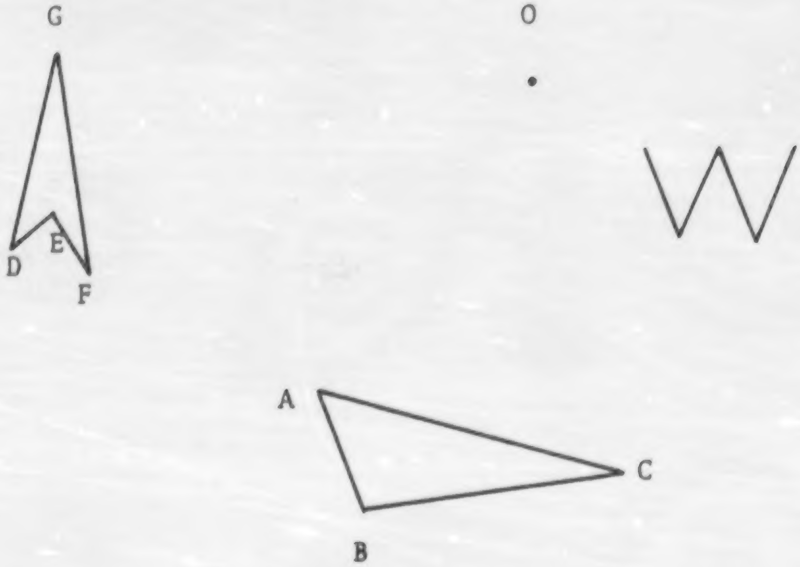


Transparency 3
Unit VIII, Set 3



From Geometry by Ruth E. M. Wong. Honolulu, HI: Research Corporation of the University of Hawaii, 1980, p. 213.

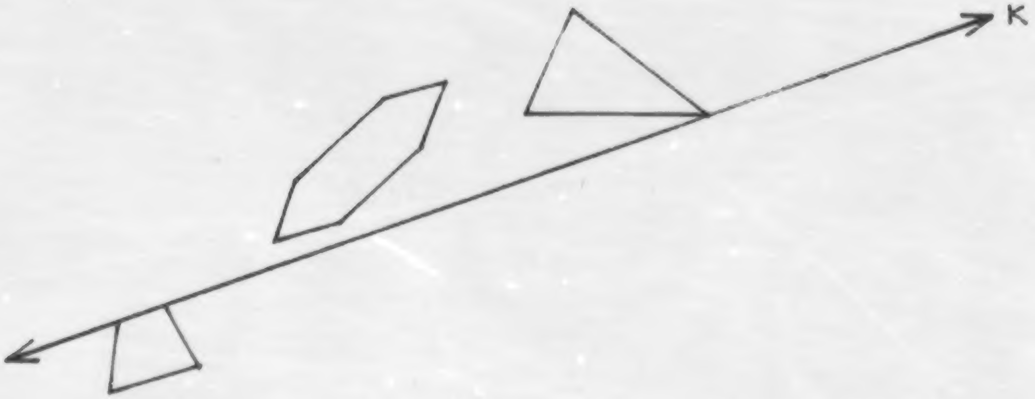
Transparency 4
Unit VIII, Set 3



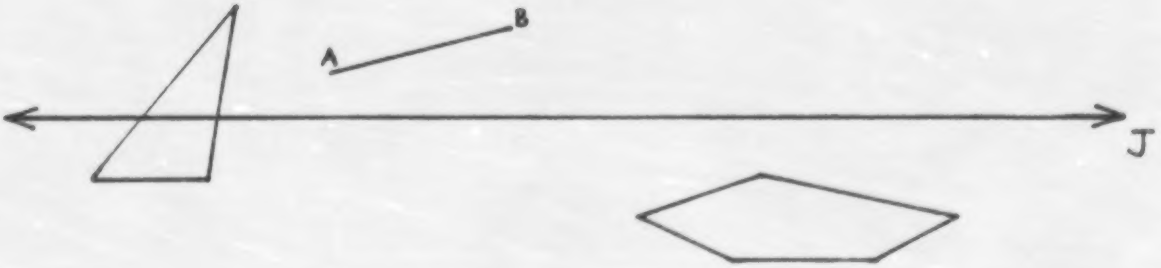
Unit VIII, Set 1
Supplementary Worksheet 1

Name____________________

Find the images of the figures under a reflection in line K.



Find the images of the figures under a reflection in line J.

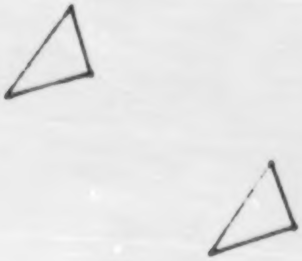


Unit VIII, Set 1
Supplementary Worksheet 2

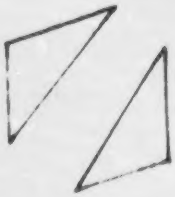
Name ____________________

Circle the reflection congruent figures. Then draw in the axes of reflections.

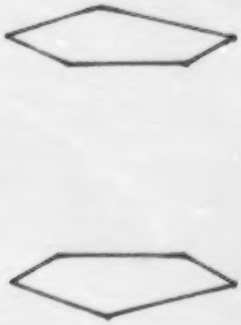
1.



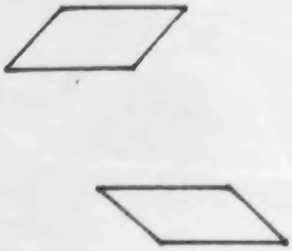
4.



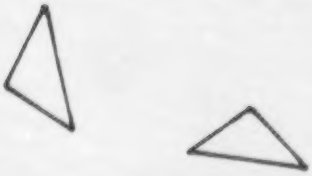
2.



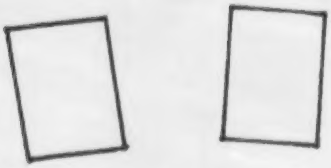
5.



3.



6.

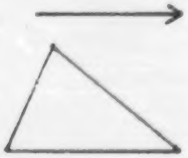


Unit VIII, Set 2
Supplementary Worksheet 3

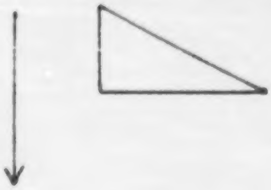
Name______________________________

Find the image of the given figure under the given translation.

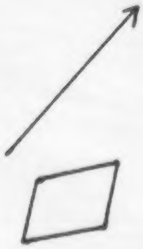
1.



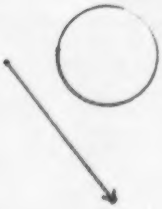
2.



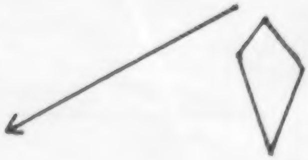
3.



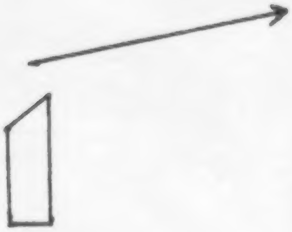
4.



5.



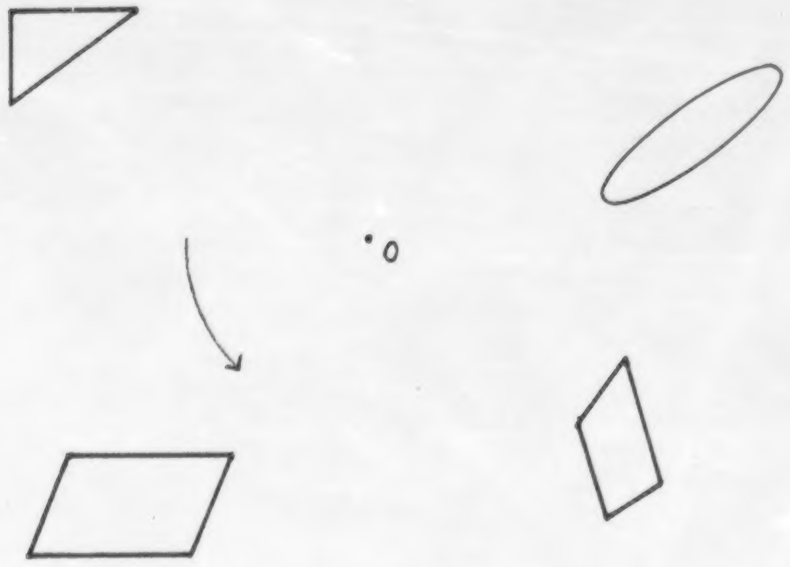
6.



Unit VIII, Set 3
Supplementary Worksheet 4

Name______________________________

Find the images of the figures under the given rotation.

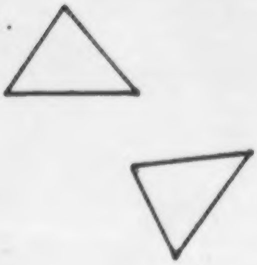


Unit VII, Set 3
Supplementary Worksheet 5

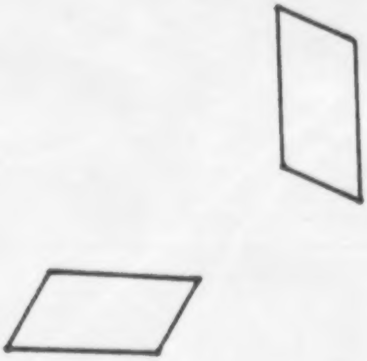
Name ______________________________

Circle the rotation congruent figures. Then find the center of rotation and draw the directed arc.

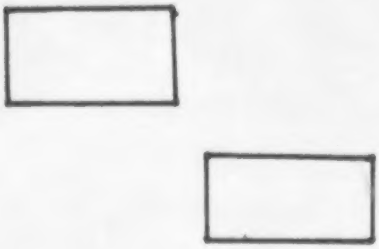
1.



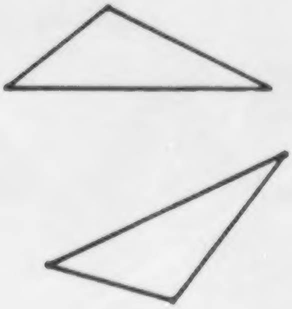
2.



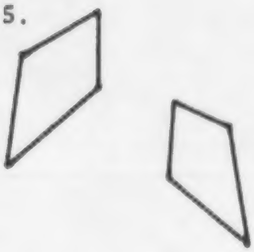
3.



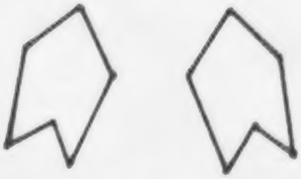
4.



5.



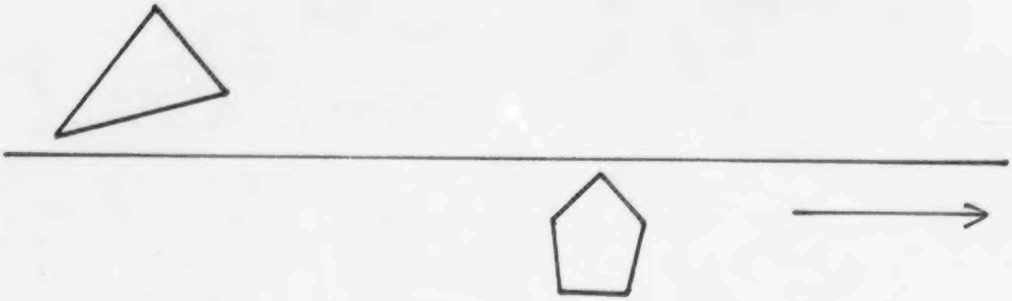
6.



Unit VIII, Set 4
Supplementary Worksheet 6

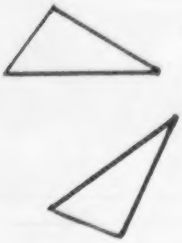
Name________________________

1. Find the images of the figures under the given glide reflection.

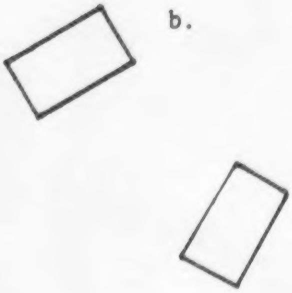


2. Decide which of the congruent pairs are related by a glide reflection.

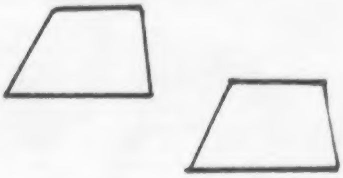
a.



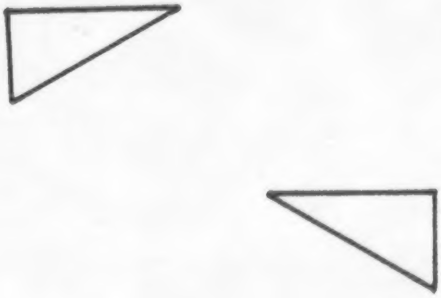
b.



c.



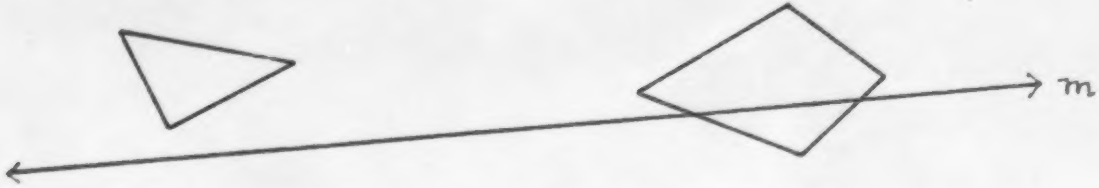
d.



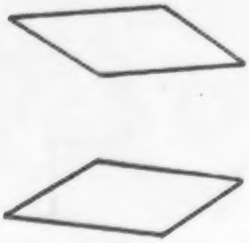
Unit VIII
Quiz 1

Name ______________________

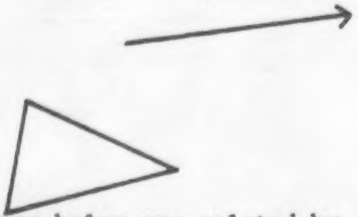
1. Find the image of each figure under a reflection in line m:



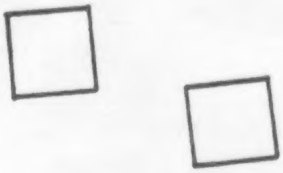
2. Draw the axis of reflection for the reflection congruent figures below:



3. Find the image of the given figure under the given translation.



4. The figures below are related by a translation. Draw an arrow to represent the translation.

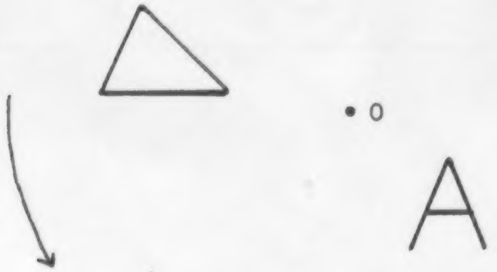


5. How are congruent figures related by a translation different from reflection congruent figures?

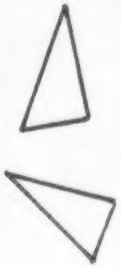
Unit VIII
Quiz 2

Name ______________________

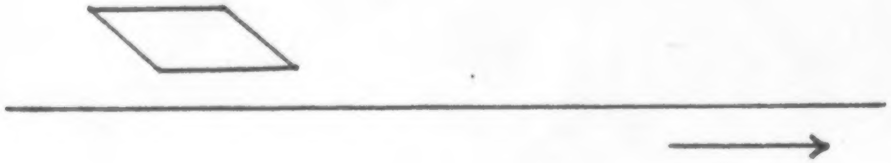
1. Find the image of each figure under the given rotation.



2. Find the center of rotation and draw the directed arc for the pair of rotation congruent figures below.



3. Find the image of the figure under the given glide reflection.

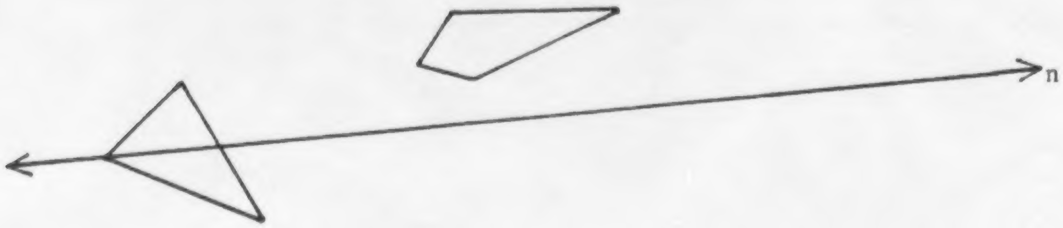


4. How are congruent figures related by a rotation different from glide reflection congruent figures?

Unit VIII
Review Worksheet

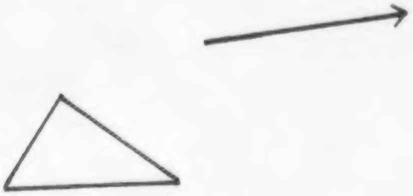
Name______________________________

1. Find the image of each figure under a reflection in line n:

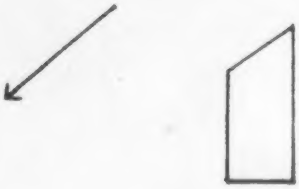


2. Find the image of each figure under the given translation.

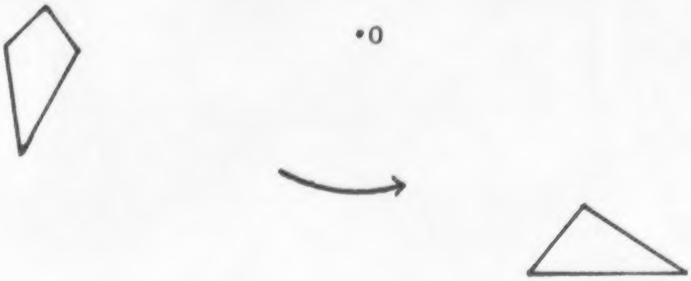
a.



b.

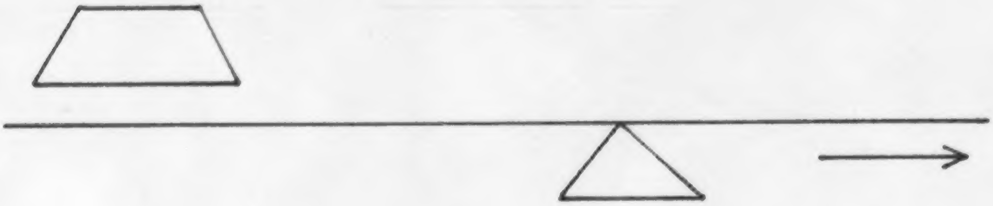


3. Find the image of each figure under the given rotation.



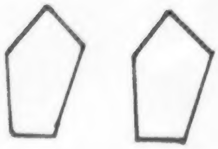
(Go on to the next page.)

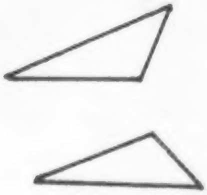
4. Find the image of each figure under a glide reflection.

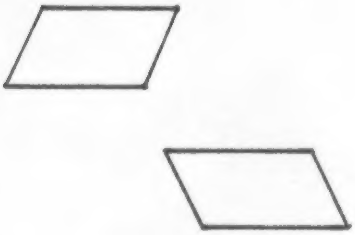


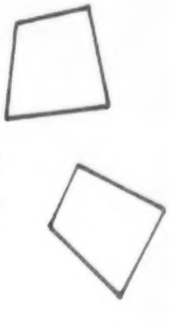
5. a. Decide whether the congruent pairs below are related by a reflection, a translation, a rotation, or a glide reflection.

   b. 1) If the figures are related by a reflection, draw in the axis of reflection.
      2) If the figures are related by a translation, draw an arrow to represent the translation.
      3) If the figures are related by rotation, find the center of rotation and draw the directed arc.





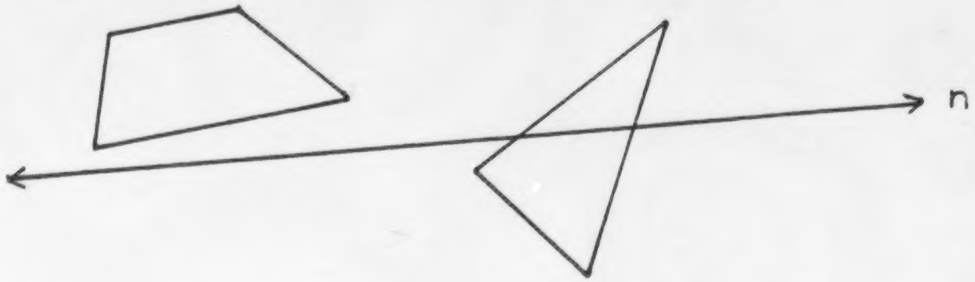




Unit VIII Test Name ______________________________

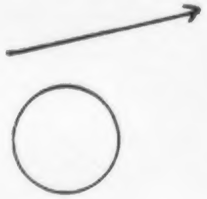
Materials needed: pencil, straightedge, compass, and tracing paper

1. Find the image of each figure under a reflection in line n.

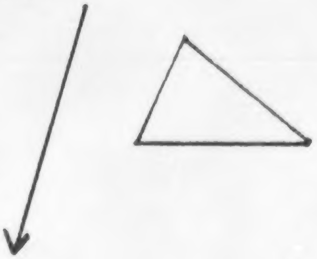


2. Find the image of each figure under the given translation.

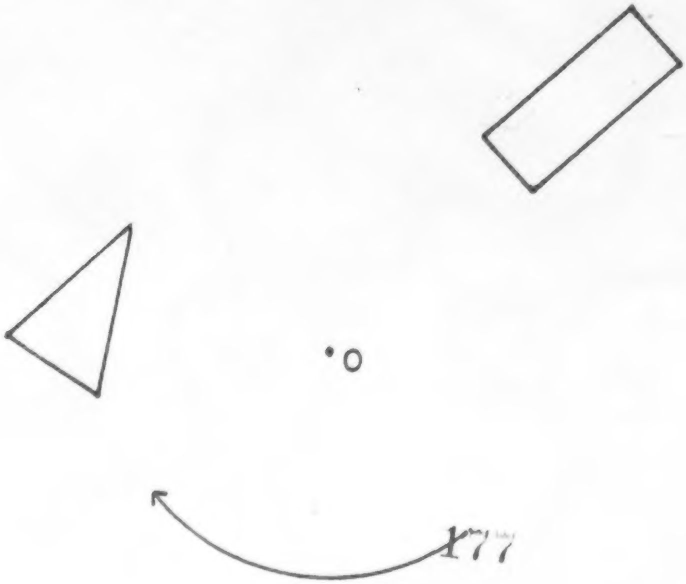
a.



b.

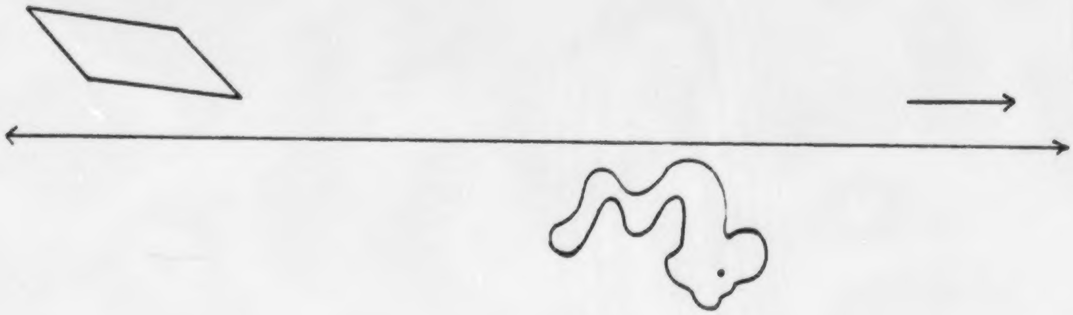


3. Find the image of each figure under the given rotation.



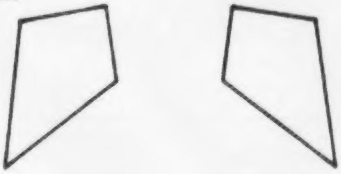
(Go on to the next page.)

4. Find the images of the given figures under the given glide reflection.

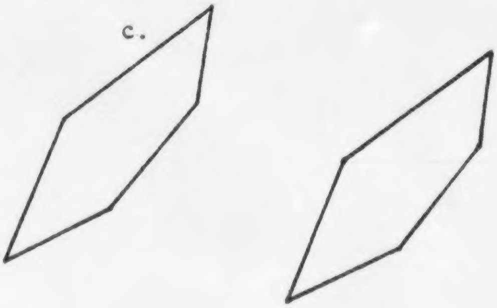


5. The congruent pairs below are related by a reflection, a translation, OR a rotation. If the figures are reflection congruent, draw the axis of reflection. If the figures are related by a translation, draw an arrow, to represent the translation. If the figures are related by a rotation, find the center of rotation and draw the directed arc.

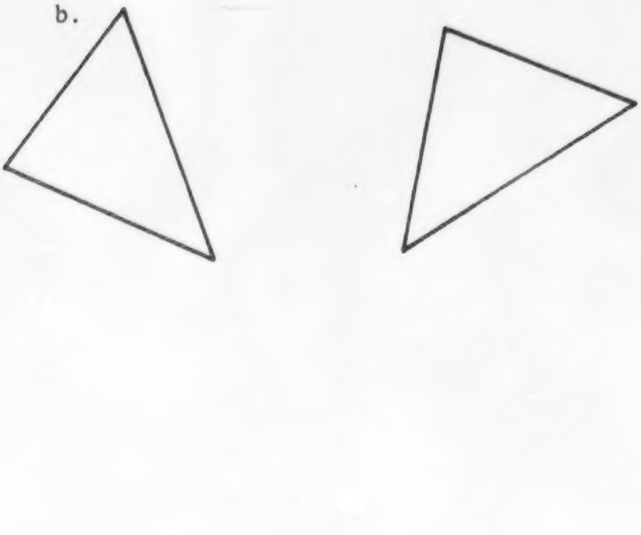
a.



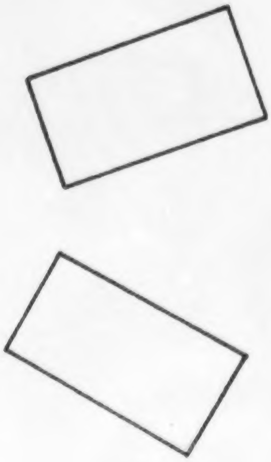
c.



b.

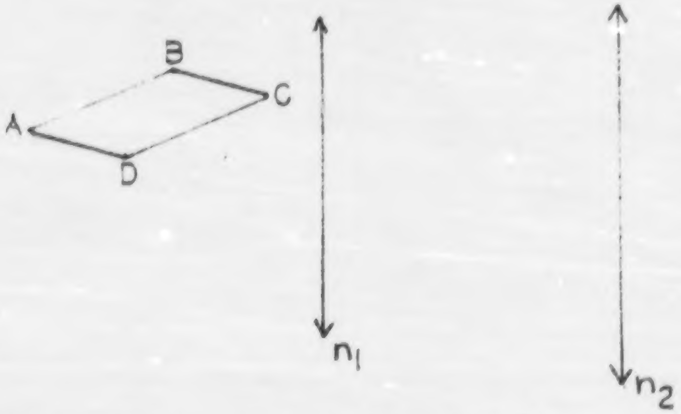


d.



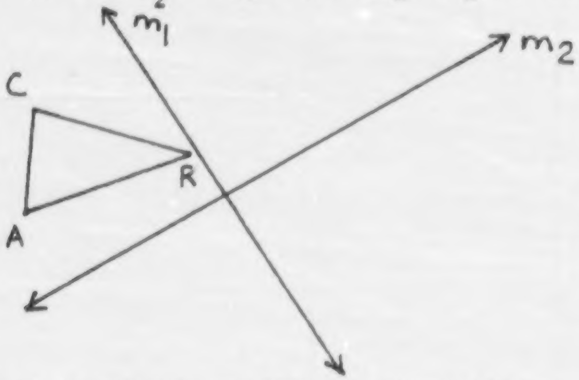
(Go on to the next page.)

6. a. Find the image of figure ABCD under a reflection in line n_1. Label the image, EFGH. Then find the image of figure EFGH under a reflection in line n_2. Label this image, IJKL.



b. Are figures ABCD and IJKL related by a translation, a rotation, or a glide reflection?

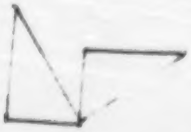
7. a. Find the image of figure CAR under a reflection in line m_1. Label the image, figure BOX. Then find the image of figure BOX under a reflection in line m_2. Label the image figure PEN.



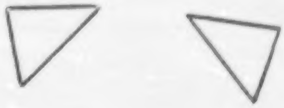
b. Are figures CAR and PEN related by a translation, a rotation, or a glide reflection?

(Go on to the next page.)

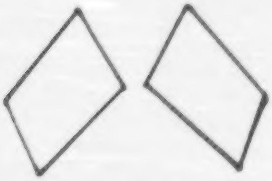
8. Explain why the figures below are not related by a glide reflection.



9. Explain why the figures below are not rotation congruent.



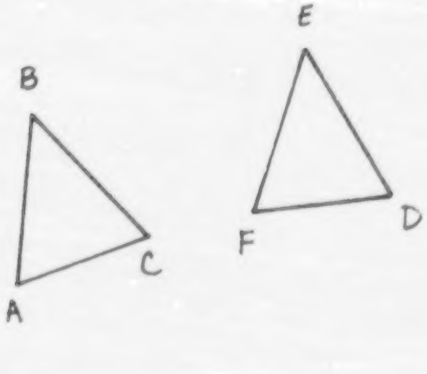
10. Explain why the figures below are not related by a translation.



11. Two congruent isosceles triangles are shown below.

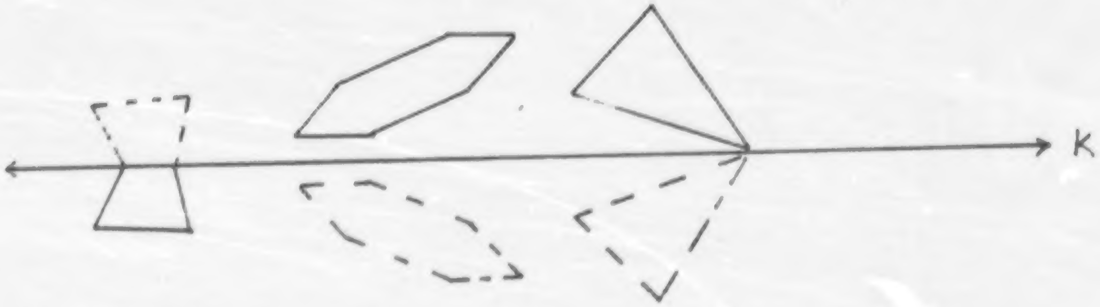
a. What transformation relates triangles ABC and DEF?

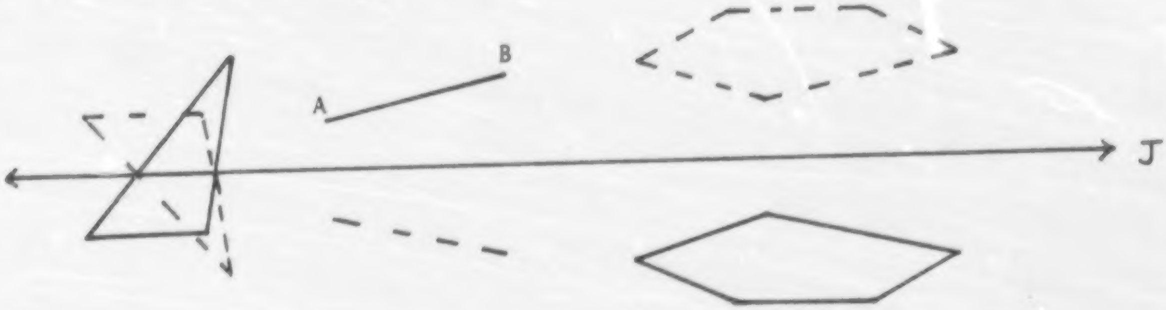
b. What transformation relates triangles ABC and FED?



ANSWER KEY

Supplementary Worksheet 1





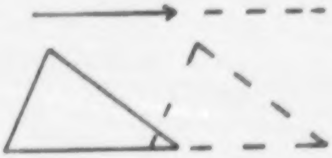
Supplementary Worksheet 2

Exercises 2, 4, and 6 should be circled and the axes of reflections drawn in.

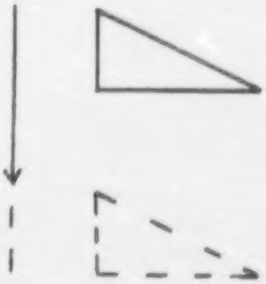
Supplementary Worksheet 3

The images of the figures are drawn in dashed lines.

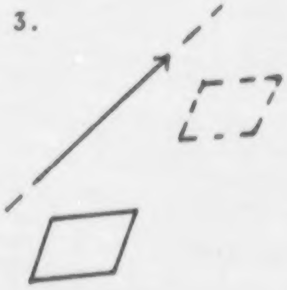
1.



2.

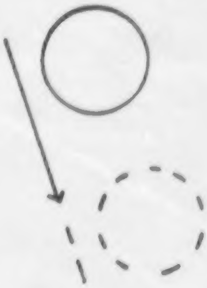


3.

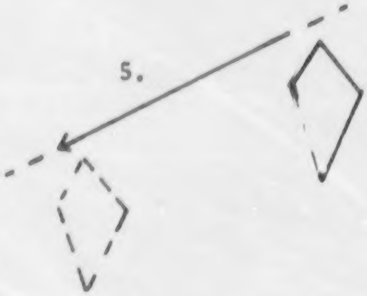


(Go on to the next page.)

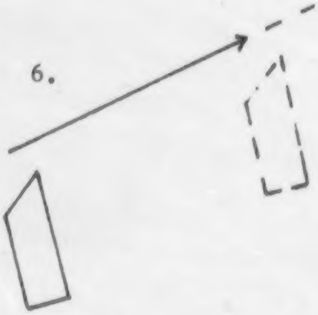
4.



5.

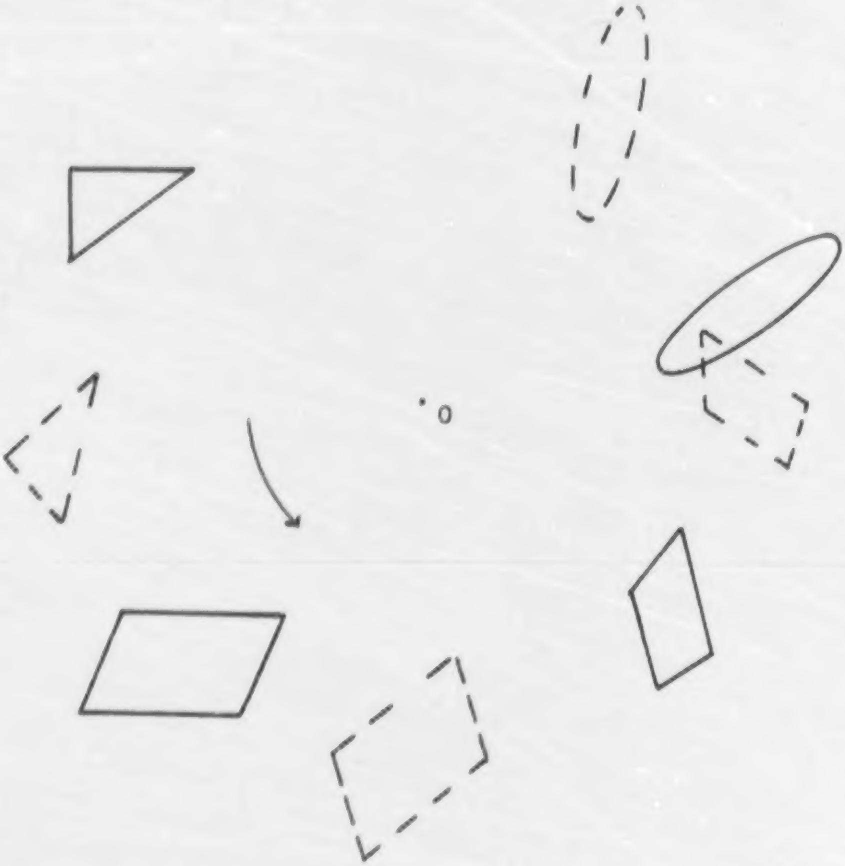


6.



Supplementary Worksheet 4

The images of the figures are drawn in dashed lines.

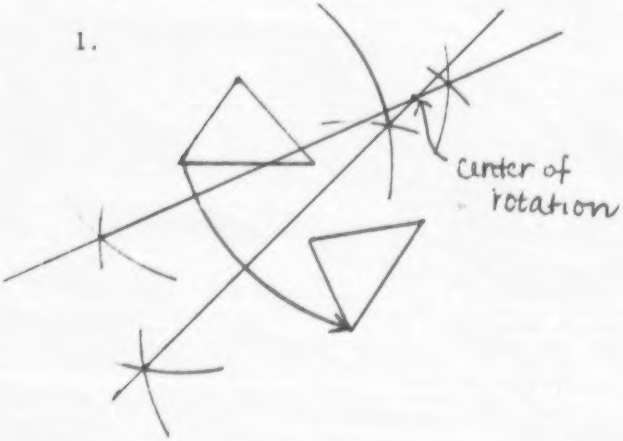


(Go on to the next page.)

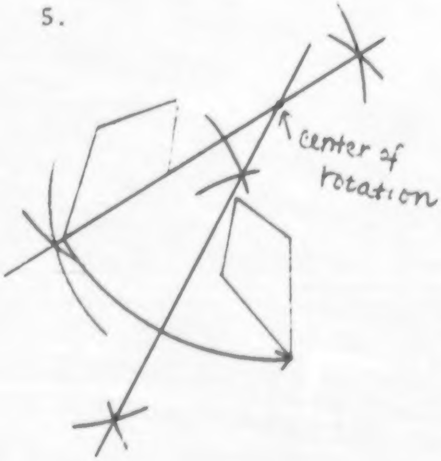
Supplementary Worksheet 5

Exercises 1 and 5 should be circled and the center of rotation and directed arc drawn:

1.

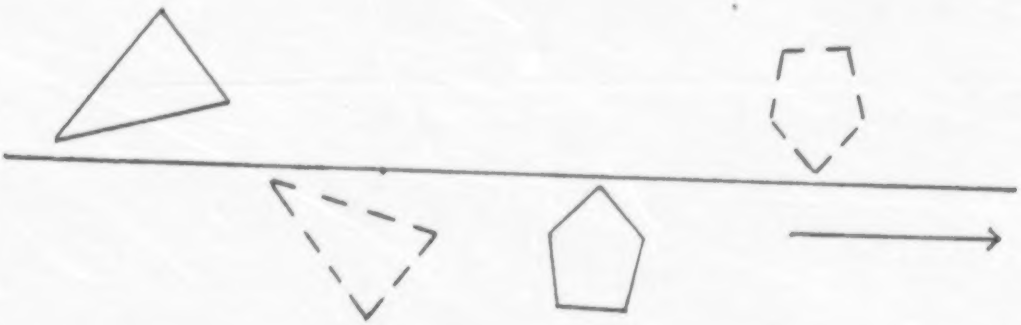

center of rotation

5.


center of rotation

Supplementary Worksheet 6

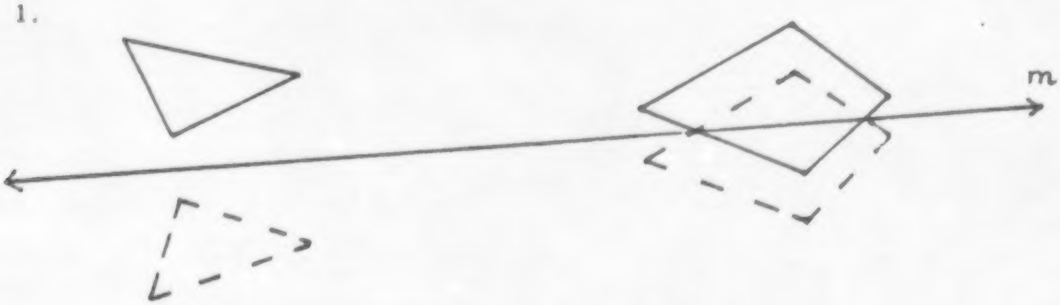
1. Images of the figures are drawn in dashed lines.



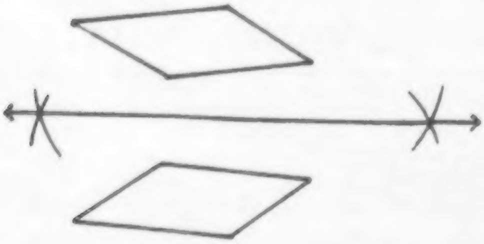
2. The pairs of figures in (d) are related by a glide reflection.

Quiz 1 *(10 points)*

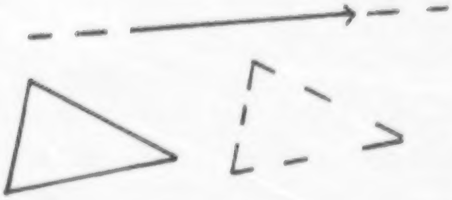
(2) 1.



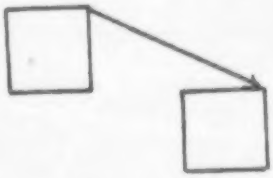
(2) 2.



(2) 3.



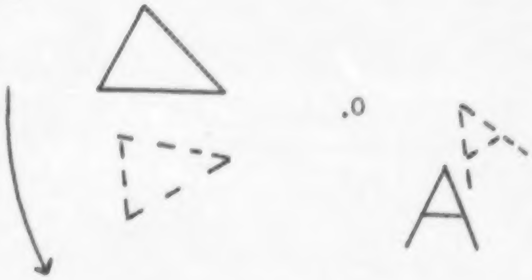
(2) 4.



(2) 5. translation congruent figures: same orientation, no fixed points
reflection congruent figures: opposite orientation, there may be fixed points

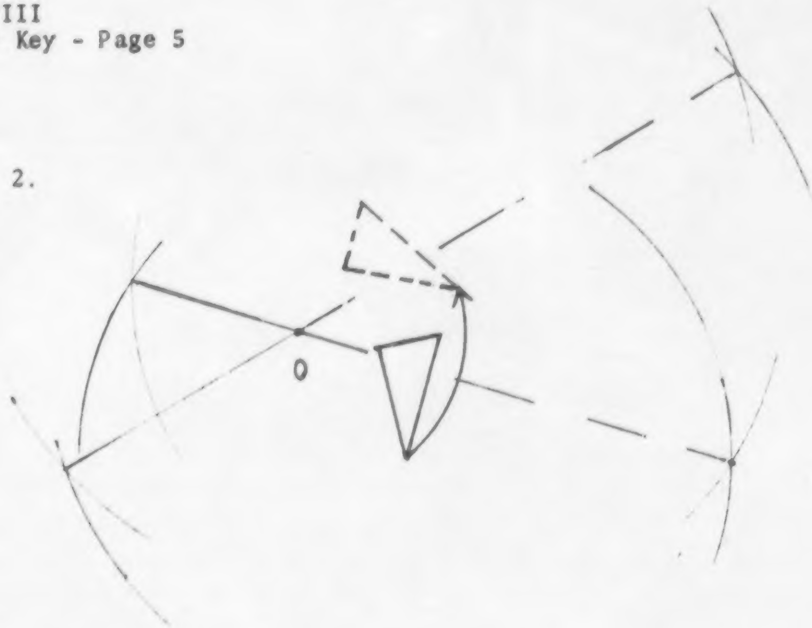
Quiz 2 *(10 points)*

(4) 1.

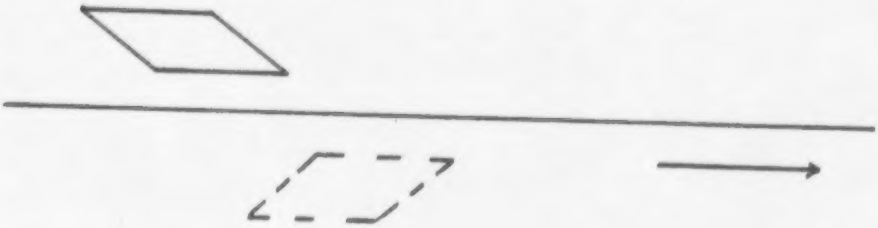


(Go on to the next page.)

(2) 2.



(2) 3.



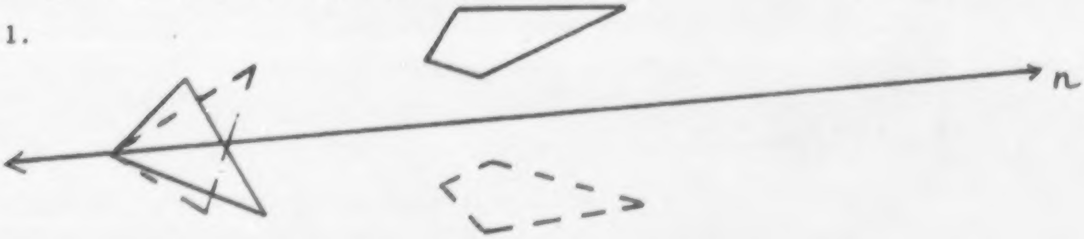
(2) 4. rotation congruent figures: same orientation, there may be fixed points

glide reflection congruent figures: opposite orientation

Review Worksheet

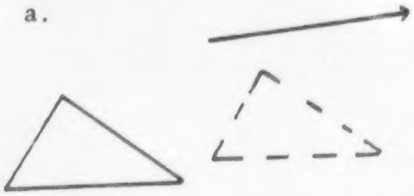
Images are shown in broken lines.

1.

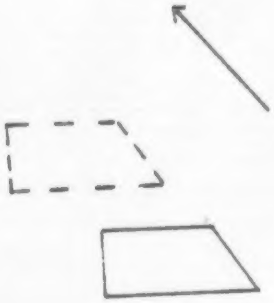


(Go on to the next page.)

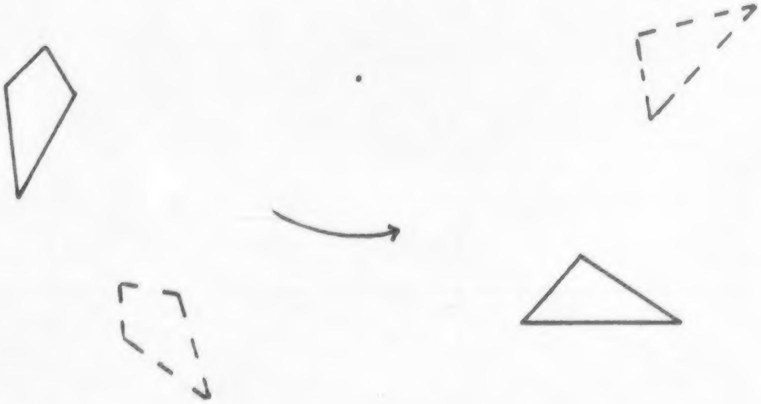
2. a.



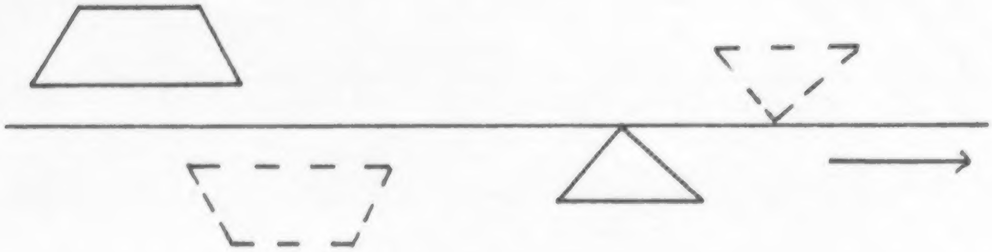
b.



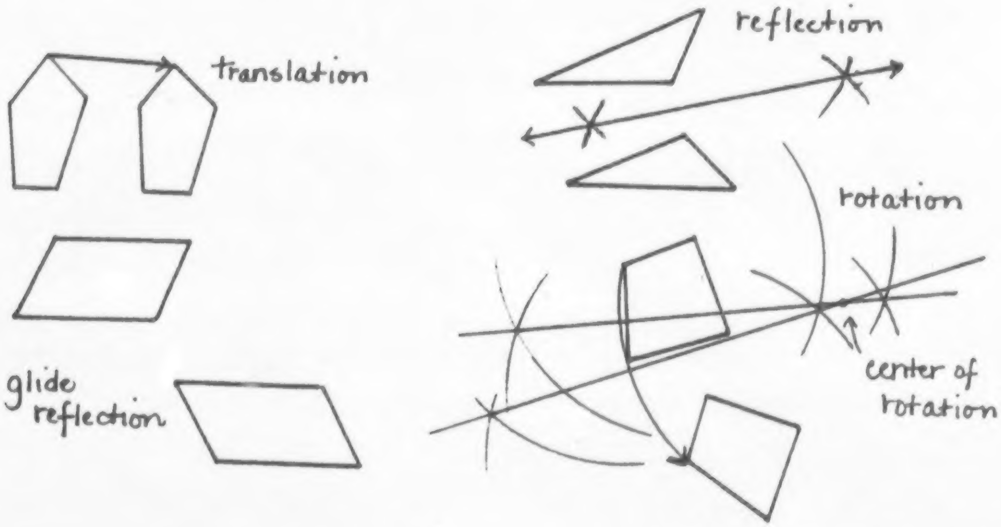
3.



4.



5.

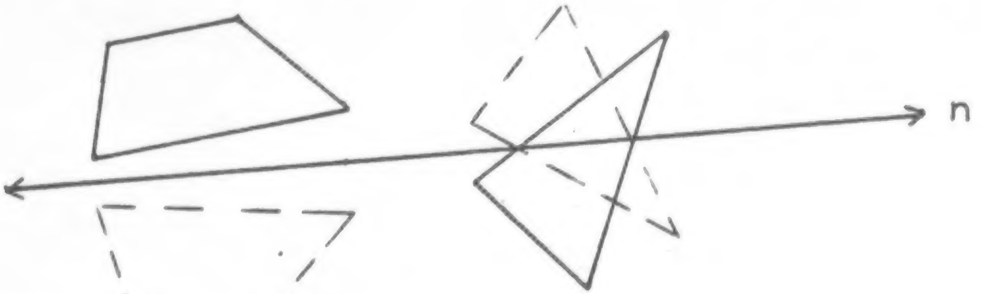

translation
reflection
rotation
center of rotation
glide reflection

(Go on to the next page.)

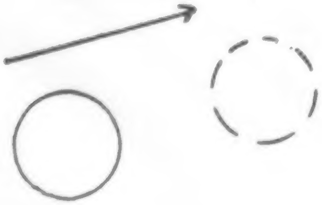
Unit VIII Test (*50 points*)

Images are shown in dashed lines.

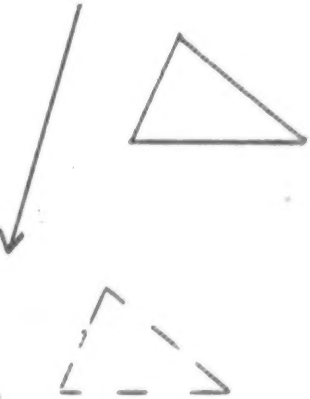
1.



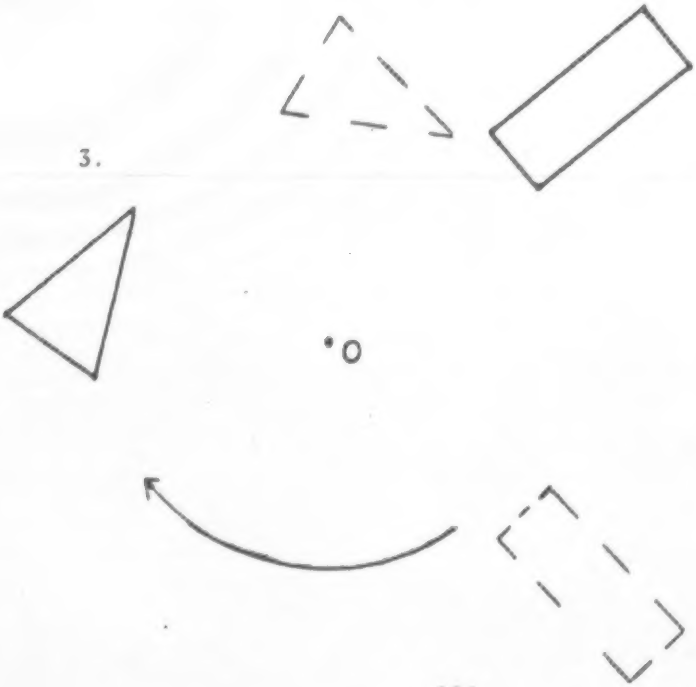
2. a.



b.



3.



(Go on to the next page.)